Chapter 2

Static Spherical Symmetry

In this chapter the theory of gravitation in flat space-time stated in the previous chapter I is applied to static spherically symmetric problems with the matter tensor of a perfect fluid.

It is useful to introduce spherical polar coordinates (r,ϑ,ϕ) with

$$x^1 = r\sin\vartheta\cos\phi,\ x^2 = r\sin\vartheta\sin\phi,\ x^3 = r\cos\vartheta. \tag{2.1}$$

We get by simple computations

$$\eta_{11} = 1,\ \eta_{22} = r^2,\ \eta_{33} = r^2\sin^2\vartheta,\ \eta_{44} = -1,\ \eta_{ij} = 0\ (i \neq j). \tag{2.2}$$

Then, we have

$$(-\eta)^{1/2} = r^2\sin\vartheta.$$

The non-vanishing Christoffel symbols of the metric are

$$\begin{gathered}\Gamma^1{}_{22} = -r, \Gamma^2{}_{12} = \Gamma^2{}_{21} = \frac{1}{r},\ \Gamma^3{}_{13} = \Gamma^3{}_{31} = \frac{1}{r},\ \Gamma^1{}_{33} = -r\sin^2\vartheta, \\ \Gamma^2{}_{33} = -\sin\vartheta\cos\vartheta,\ \Gamma^3{}_{23} = \Gamma^3{}_{32} = \cot\vartheta\end{gathered}. \tag{2.3}$$

2.1 Field Equations, Equations of Motion and Energy-Momentum

The potentials are written in the form

$$\begin{gathered}g^{11} = f(r), g^{22} = \frac{g(r)}{r^2}, g^{33} = \frac{g(r)}{r^2\sin^2\vartheta}, \\ g^{44} = -h(r), g^{ij} = 0, (i \neq j)\end{gathered} \tag{2.4a}$$

It follows

$$\begin{gathered}g_{11} = \frac{1}{f}, g_{22} = \frac{r^2}{g}, g_{33} = \frac{r^2\sin^2\vartheta}{g}, \\ g_{44} = -\frac{1}{h}, g_{ij} = 0, (i \neq j)\end{gathered} \tag{2.4b}$$

We get

$$(-G)^{1/2} = \frac{r^2 \sin\vartheta}{g(fh)^{1/2}}, \left(\frac{-G}{-\eta}\right)^{1/2} = \frac{1}{g(fh)^{1/2}}, \tag{2.5}$$

For a body at rest we have $u^1 = u^2 = u^3 = 0,$ i.e. it follows from relation (1.13)

$$u^4 = ch^{1/2}. \tag{2.6}$$

Then, the matter tensor of a perfect fluid (1.28) is given by

$$\begin{aligned} T(M)^i_{\ j} &= pc^2, (i = j = 1,2,3) \\ &= -\rho c^2, (i = j = 4) \\ &= 0. (i \neq j) \end{aligned} \tag{2.7}$$

We get from the equations (1.21a) and (1.9) by the use of (2.4) and (2.5) the energy-momentum tensor of the gravitational field

$$\begin{aligned} T(G)^i_{\ j} &= -\frac{1}{16\kappa}(L_1 - L_2), (i = j = 1) \\ &= \frac{1}{16\kappa} L_1, (i = j = 2,3) \\ &= \frac{1}{16\kappa}(L_1 + L_2), (i = j = 4) \\ &= 0, (i \neq j) \end{aligned} \tag{2.8}$$

Here,

$$L_1 = -\frac{f}{g(fh)^{1/2}}\left(\left(\frac{f'}{f}\right)^2 + 2\left(\frac{g'}{g}\right)^2 + \left(\frac{h'}{h}\right)^2 - \frac{1}{2}\left(\frac{f'}{f} + 2\frac{g'}{g} + \frac{h'}{h}\right)^2\right), \tag{2.9a}$$

$$L_2 = -\frac{4}{r^2}\frac{f}{g(fh)^{1/2}}\left(\frac{f-g}{f}\right)^2, \tag{2.9b}$$

$$L = L_1 + L_2, \tag{2.9c}$$

where the prime $'$ denotes differentiation with regard to the distance r. The field equations (1.24) with $\Lambda = 0$ give by the use of the covariant derivatives the following three equations:

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{f}{g(fh)^{1/2}}\frac{f'}{f}\right)-\frac{2}{r^2}\frac{f}{g(fh)^{1/2}}\frac{f^2-g^2}{f^2}=-\frac{1}{2}L_1+2\kappa c^2(\rho-p), \quad (2.10a)$$

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{f}{g(fh)^{1/2}}\frac{g'}{g}\right)+\frac{1}{r^2}\frac{f}{g(fh)^{1/2}}\frac{f^2-g^2}{f^2}=-\frac{1}{4}L_2+2\kappa c^2(\rho-p), \quad (2.10b)$$

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{f}{g(fh)^{1/2}}\frac{h'}{h}\right)=-2\kappa c^2(\rho+3p). \quad (2.10c)$$

The conservation law of the energy-momentum (1.25a) implies

$$\frac{d}{dr}(L_2-L_1)-\frac{4}{r}L_2+16\kappa c^2\frac{d}{dr}p=0.$$

It follows by multiplication with r^3

$$\frac{d}{dr}\left[r^3(L_2-L_1)\right]=r^2(L_1+L_2)-16\kappa c^2 r^3\frac{d}{dr}p. \quad (2.11)$$

The equations of motion (1.29a) yield

$$\frac{d}{dr}p=-\frac{1}{2}\left(\frac{f'}{f}+2\frac{g'}{g}\right)p+\frac{1}{2}\frac{h'}{h}\rho. \quad (2.12)$$

In addition to the equations (2.10), (2.11) and (2.12) we have an equation of state

$$p=p(\rho). \quad (2.13)$$

The natural boundary conditions are for $r\to\infty$

$$f(r)\to 1,\ g(r)\to 1,\ h(r)\to 1 \quad (2.14a)$$

and for $r\to 0$

$$r^2\frac{f}{g(fh)^{1/2}}\frac{f'}{f}\to 0,\ r^2\frac{f}{g(fh)^{1/2}}\frac{g'}{g}\to 0,\ r^2\frac{f}{g(fh)^{1/2}}\frac{h'}{h}\to 0. \quad (2.14b)$$

2.2 Gravitational and Inertial Mass

Let us assume a spherically symmetric star with radius r_0 . Then, the boundary condition of the pressure has the form

$$p(r_0)=0\,. \tag{2.15}$$

The mass and the pressure are defined by

$$M=4\pi\int_0^{r_0} r^2\rho(r)\,dr\,,\ P=4\pi\int_0^{r_0} r^2 p(r)\,dr\,. \tag{2.16}$$

We get from (2.10c) with the aid of the boundary conditions (2.14) for $r>r_0$

$$r^2\frac{f}{g(fh)}\frac{h'}{h}=-2\frac{k(M+3P)}{c^2} \tag{2.17}$$

where (1.14) is used. Equation (2.17) gives by integration and the boundary conditions (2.14) for $r>r_0$

$$h(r)=1+2\frac{k(M+3P)}{c^2}\frac{1}{r}+O\left(\frac{1}{r^2}\right). \tag{2.18}$$

Equation (2.18) implies the gravitational mass

$$M_g=M+3P\,. \tag{2.19}$$

The inertial mass M_i is given by

$$M_i c^2=-4\pi\int\left(T(M)^4{}_4+T(G)^4{}_4\right)r^2dr \tag{2.20}$$

It follows by the use of (2.7), (2.8) and (2.16)

$$M_i=M-\frac{c^2}{16k}\int_0^{\infty} r^2\left(L_1+L_2\right)dr\,. \tag{2.21}$$

We put by virtue of (2.14a) and $r>>r_0$

$$f = 1 - 2\frac{\alpha}{r} + O\left(\frac{1}{r^2}\right), g = 1 - 2\frac{\beta}{r} + O\left(\frac{1}{r^2}\right). \tag{2.22}$$

Equation (2.10) gives by integration and the use of (2.14b)

$$r^2 \frac{f}{g(fh)^{1/2}} \frac{g'}{g} + 2\int_0^r \frac{f}{g(fh)^{1/2}} \frac{(f-g)g}{f^2} dr = 2\kappa c^2 \int_0^r r^2 (\rho - p) dr .$$

It follows for $r \to \infty$ with the aid of (2.18), (2.22) and (2.16)

$$\int_0^\infty \frac{f}{g(fh)^{1/2}} \frac{(f-g)g}{f^2} dr = \frac{k}{c^2}(M - P) - \beta . \tag{2.23}$$

The existence of the integral of equation (2.23) gives by using (2.18) and (2.22) $\alpha = \beta$,i.e., we have

$$f = 1 - 2\frac{\alpha}{r} + O\left(\frac{1}{r^2}\right), \; g = 1 - 2\frac{\alpha}{r} + O\left(\frac{1}{r^2}\right). \tag{2.24}$$

We assume the natural boundary conditions as $r \to \infty$

$$r^3 L_1 \to 0, \quad r^3 L_2 \to 0, \quad r^3 p \to 0 .$$

Then, equation (2.11) implies by integration

$$\begin{aligned} r^3 (L_2 - L_1) = \int_0^r r^2 (L_1 + L_2) dr - 16\kappa c^2 r^3 p(r) \\ +48\kappa c^2 \int_0^r r^2 p(r) dr. \end{aligned} \tag{2.25}$$

Hence, we get for $r \to \infty$ by the use of (2.18), (2.24), (2.15) and (2.16)

$$\int_0^\infty r^2 (L_1 + L_2) dr = -48\frac{k}{c^2} P . \tag{2.26}$$

Substituting equation (2.26) into (2.21) it follows with equation (2.19)

$$M_i = M + 3P = M_g , \tag{2.27}$$

i.e., inertial and gravitational mass are identical.

In general relativity the definition of inertial mass gives difficulties by virtue of the non-covariance of the energy-momentum of the gravitational field (see e.g. [Dem 82]).

In particular, equation (2.26) can be rewritten

$$-\frac{4\pi}{c^2}\int_0^\infty r^2 T\left(G\right)^4{}_4 dr = 3P\,. \tag{2.28}$$

Equation (2.12) together with (2.10c) implies that there exists no spherically symmetric star without pressure.

We get by a suitable linear combination of the equations (2.10) and by integration using the boundary conditions (2.14b)

$$\begin{aligned} &r^2 \frac{f}{g\left(fh\right)^{1/2}}\left(\frac{f'}{f}+2\frac{g'}{g}+3\frac{h'}{h}\right) \\ &=-\frac{1}{2}\int_0^r r^2\left(L_1+L_2\right)dr-24kc^2\int_0^r r^2 p\left(r\right)dr. \end{aligned} \tag{2.29}$$

Hence, we have for $r\to\infty$ by virtue of (2.26), (2.17) and (2.24)

$$\alpha=\frac{k}{c^2}\left(M+3P\right)=\frac{k}{c^2}M_g\,. \tag{2.30}$$

Put

$$K=\frac{kM_g}{c^2} \tag{2.31}$$

then, we have for $r>>r_0$

$$\begin{aligned} f=1-2\frac{K}{r}+O\left(\left(\frac{K}{r}\right)^2\right),\ g=1-2\frac{K}{r}+O\left(\left(\frac{K}{r}\right)^2\right), \\ h=1+2\frac{K}{r}+O\left(\left(\frac{K}{r}\right)^2\right). \end{aligned} \tag{2.32}$$

Equation (2.23) gives

$$\int_0^\infty \frac{f}{g(fh)^{1/2}} \frac{(f-g)g}{f^2} dr = -4\frac{k}{c^2} P . \tag{2.33}$$

The gravitational field in the exterior of the spherically symmetric star with pressure is given to the first order approximation by (2.32), i.e. by one mass, namely the gravitational mass M_g. This is similar to Einstein's general theory of relativity in contrast to Rosen's bi-metric gravitation theory where the field is described by two mass parameters M_g and M' with $M_g \neq M'$ for non-vanishing pressure.

2.3 Gravitational Field in the Exterior

Let us study the gravitational field in the exterior of the star, i.e. $r > r_0$. We have from (2.10a), (2.10b) and (2.17) with the definitions (2.19) and (2.31)

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2 \frac{f}{g(fh)^{1/2}} \frac{f'}{f} \right) - \frac{2}{r^2} \frac{f}{g(fh)^{1/2}} \frac{f^2-g^2}{f^2} = -\frac{1}{2} L_1 \tag{2.34a}$$

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2 \frac{f}{g(fh)^{1/2}} \frac{g'}{g} \right) + \frac{1}{r^2} \frac{f}{g(fh)^{1/2}} \frac{f^2-g^2}{f^2} = -\frac{1}{4} L_2 \tag{2.34b}$$

$$r^2 \frac{f}{g(fh)^{1/2}} \frac{h'}{h} = -2K . \tag{2.34c}$$

Substituting

$$\xi = K / r$$

into equations (2.34) we get by elementary computations

$$\frac{d}{d\xi}\left(\frac{f_\xi}{f} \right) = \frac{2}{\xi^2}\left(1 - \left(\frac{g}{f} \right)^2 \right) - \frac{1}{4}\left(\frac{f_\xi}{f} \right)^2 + \frac{1}{4}\left(\frac{h_\xi}{h} \right)^2 - \frac{g_\xi}{g}\frac{h_\xi}{h} \tag{2.35a}$$

$$\frac{d}{d\xi}\left(\frac{g_\xi}{g}\right)=-\frac{2}{\xi^2}\left(1-\frac{g}{f}\right)\frac{g}{f}+\left(\frac{g_\xi}{g}\right)^2-\frac{1}{2}\frac{f_\xi}{f}\frac{g_\xi}{g}+\frac{1}{2}\frac{g_\xi}{g}\frac{h_\xi}{h} \tag{2.35b}$$

$$\frac{h_\xi}{h}=2g\left(\frac{h}{f}\right)^{1/2} \tag{2.35c}$$

where, the index ξ means the derivative relative to ξ. Put

$$f=\exp(x+z), g=\exp(y+z), h=\exp(-z). \tag{2.36}$$

Then, it follows from (2.35)

$$x_{\xi\xi\xi}=\frac{2}{\xi^2}\left(1-\exp\left(2(y-x)\right)\right)+4\exp(2y-x)-\frac{1}{4}\left(x_\xi\right)^2 \tag{2.37a}$$

$$y_{\xi\xi\xi}=-\frac{2}{\xi^2}\left(1-\exp(y-x)\right)\exp(y-x)-\frac{1}{2}x_\xi y_\xi+\left(y_\xi\right)^2 \tag{2.37b}$$

$$z_\xi=-2\exp(y-x/2). \tag{2.37c}$$

The equations (2.35) and (2.32) imply for $\xi\to 0$

$$x=O\left(\xi^3\right), y=O\left(\xi^2\right), z=-2\xi+O\left(\xi^2\right).$$

Substituting the approximations of x and y up to the order four in ξ into the equations (2.37a) and (2.37b) we get by elementary calculations

$$x\approx 2A\xi^3+\frac{1}{6}\xi^4,\ y\approx\xi^2-A\xi^3+\frac{2}{3}\xi^4 \tag{2.38a}$$

where A is an arbitrary parameter which must be fixed by the interior solution. Equation (2.37c) together with (2.38a) yields

$$z\approx -2\xi-\frac{2}{3}\xi^3+A\xi^4. \tag{2.38b}$$

Finally, we obtain from (2.36) and (2.38) up to order four in $\frac{K}{r}$:

$$f \approx 1-2\frac{K}{r}+2\left(\frac{K}{r}\right)^2-(2-2A)\left(\frac{K}{r}\right)^3+\left(\frac{13}{6}-3A\right)\left(\frac{K}{r}\right)^4 \tag{2.39a}$$

$$g \approx 1-2\frac{K}{r}+3\left(\frac{K}{r}\right)^2-(4+A)\left(\frac{K}{r}\right)^3+\left(\frac{31}{6}+3A\right)\left(\frac{K}{r}\right)^4 \tag{2.39b}$$

$$h \approx 1+2\frac{K}{r}+2\left(\frac{K}{r}\right)^2+2\left(\frac{K}{r}\right)^3+(2-A)\left(\frac{K}{r}\right)^4. \tag{2.39c}$$

Elementary computations give up to order five in K

$$\begin{aligned} L_1 &\approx -8\frac{K^2}{r^4}-8\frac{K^4}{r^6}+40A\frac{K^5}{r^7}, \\ L_2 &\approx -4\frac{K^4}{r^6}+24A\frac{K^5}{r^7}, \\ L_G &\approx -8\frac{K^2}{r^4}-12\frac{K^4}{r^6}+64A\frac{K^5}{r^7}. \end{aligned} \tag{2.40}$$

It is easily proved that the conservation law of energy-momentum (2.11) holds to the considered accuracy.

Einstein's theory gives in harmonic coordinates

$$f_E=\frac{1-K/r}{1+K/r}\approx 1-2\frac{K}{r}+2\left(\frac{K}{r}\right)^2-2\left(\frac{K}{r}\right)^3+2\left(\frac{K}{r}\right)^4 \tag{2.41a}$$

$$g_E=\frac{1}{(1+K/r)^2}\approx 1-2\frac{K}{r}+3\left(\frac{K}{r}\right)^2-4\left(\frac{K}{r}\right)^3+5\left(\frac{K}{r}\right)^4 \tag{2.41b}$$

$$h_E=\frac{1+K/r}{1-K/r}\approx 1+2\frac{K}{r}+2\left(\frac{K}{r}\right)^2+2\left(\frac{K}{r}\right)^3+2\left(\frac{K}{r}\right)^4. \tag{2.41c}$$

The solution in the exterior of the star by Einstein's theory does not contain a free parameter. The results of the two theories agree for f and g up to the order two and for h up to the order three in the case $A\neq 0$ and for $A=0$ the agreement of the solutions for f and g is up to the order three and for h up to the order four. Hence, we have high agreement of the exterior solutions of both theories.

We will now give a lower limit for the pressure of stars on the assumption that K/r_0 is small. Let us assume a non-negative density of the gravitational energy in the interior of the body, i.e.

$$-T\left(G\right)^4_{\ 4} \geq 0 \text{ for } r \leq r_0$$

then, it follows by the use of (2.26), (2.8) and (2.40)

$$3P = \frac{4\pi}{c^2}\int_0^\infty r^2\left(-T\left(G\right)^4_{\ 4}\right)dr \geq \frac{4\pi}{c^2}\int_{r_0}^\infty r^2\left(-T\left(G\right)^4_{\ 4}\right)dr$$
$$= \frac{c^2}{16k}\left(8\frac{K^2}{r_0} + 4\frac{K^4}{{r_0}^3} - 16A\frac{K^5}{{r_0}^4}\right).$$

Hence, we have by the use of (2.31)

$$\frac{K}{r_0} + \frac{1}{2}\left(\frac{K}{r_0}\right)^3 - 2A\left(\frac{K}{r_0}\right)^4 \leq 6\frac{P}{M_g}. \tag{2.42}$$

Inequality (2.42) gives for our Sun ($M_\oplus \approx 1.993 \bullet 10^{33} g$, $r_\oplus \approx 6.96 \bullet 10^{10} cm$)

$$P_\oplus / M_\oplus \geq 3.6 \bullet 10^{-7}.$$

Numerical methods are used to obtain the solution in the exterior of the star for large values of $\xi = K/r$. For small ξ $\left(\leq 10^{-2}\right)$ the solutions (2.38) and (2.39) are used. The system of the differential equations (2.37) is numerically solved by the use of Runge-Kutta methods for different values of the parameter A. There are two different types of solutions: (1) regular solutions, i.e. for all $\xi \geq 0$ the functions f, g and h exist and are positive. This is the case for all values $A \geq 0.2$. (2) Singular solutions, i.e. it exists a positive value ξ_c depending on A such that f, g and h do not exist or vanish at ξ_c. Case (2) arises for small positive and all negative values of A.

2.4 Non-Singular Solutions

We will now study the solution in the vicinity of the singularity

$$\frac{f_\xi}{f} \approx \frac{\alpha}{\xi_c - \xi}, \quad \frac{g_\xi}{g} \approx \frac{\beta}{\xi_c - \xi}, \quad \frac{h_\xi}{h} \approx \frac{\delta}{\xi_c - \xi} \tag{2.43}$$

with suitable constants α, β and γ. This gives near the singularity $\xi < \xi_c$

$$f \approx \frac{A_0}{(\xi_c - \xi)^\alpha}, \quad g \approx \frac{B_0}{(\xi_c - \xi)^\beta}, h \approx \frac{C_0}{(\xi_c - \xi)^\delta} \tag{2.44}$$

with some constants A_0, B_0 and C_0. We get by the substitution of (2.43) and (2.44) into the equation (2.35c)

$$\frac{\delta}{\xi_c - \xi} = 2B_0 \left(\frac{C_0}{A_0}\right)^{1/2} \frac{1}{(\xi_c - \xi)^{\beta + (\delta - a)/2}}$$

implying

$$\beta + (\delta - \alpha)/2 = 1, \quad \delta = 2B_0 (C_0 / A_0)^{1/2} > 0. \tag{2.45a,b}$$

It follows by the substitution of (2.43) and (2.44) into (2.35b) and the use of (2.45a)

$$\beta - \alpha < 1. \tag{2.45c}$$

We have from (2.35a)

$$\alpha = \frac{1}{4}\delta^2 - \frac{1}{4}\alpha^2 - \beta\delta. \tag{2.45d}$$

The equations (2.45a) and (2.45d) yield by elementary calculations

$$1 + 3\beta^2 - 4\beta - 2\alpha\beta = 0$$

Hence, we get

$$\beta \neq 0, \quad \alpha = \frac{1 + 3\beta^2 - 4\beta}{2\beta}, \quad \delta = \frac{1 - \beta^2}{2\beta}. \tag{2.46}$$

We obtain by (2.46) and (2.45c)

$$\beta > 0$$

implying by the use of (2.46) and (2.45)

$$0 < \beta < 1. \tag{2.47}$$

Hence, we have

$$\alpha = \frac{(1-3\beta)(1-\beta)}{2\beta},\ \delta = \frac{1-\beta^2}{2\beta},\ 0 < \beta < 1 \tag{2.48a}$$

$$B_0 \left(\frac{C_0}{A_0} \right)^{1/2} = \frac{\delta}{2}. \tag{2.48b}$$

Therefore, the constants β and δ are always positive whereas α is positive for $\beta < 1/3$, negative for $\beta > 1/3$ and zero for $\beta = 1/3$. The radial velocity of light v_L near the critical value ξ_c is given by

$$v_l = c\left(\frac{f}{h} \right)^{1/2} \approx c\left(\frac{A_0}{C_0} \right)^{1/2} (\xi_c - \xi)^{1-\beta} \to 0 \tag{2.49}$$

for $\xi \to \xi_c$ by the use of (2.48a).

The solutions (2.44) cannot be continued to $\xi > \xi_c$ by virtue of (2.48a). This is similar to Rosen's bi-metric theory of gravitation $[Ros74]$. Therefore, static spherically symmetrical stars with radius $r_0 < K / \xi_c = r_c$ cannot exist in this gravitational theory.

We will now study a static spherically symmetric star with the radius $r_0 = r_c$. We get from (2.43), (2.44) and (2.48) for $r \to r_c$

$$r^2 \frac{f}{g(fh)^{1/2}} \frac{f'}{f} \to -2\frac{kM_g}{c^2} \frac{1+3\beta^2-4\beta}{1-\beta^2}$$

$$r^2 \frac{f}{g(fh)^{1/2}} \frac{g'}{g} \to -2\frac{kM_g}{c^2} \frac{2\beta^2}{1-\beta^2}$$

$$r^2 \frac{f}{g(fh)^{1/2}} \frac{h'}{h} \to -2\frac{kM_g}{c^2}.$$

Therefore, we have as $r \to r_c$

$$r^2 \frac{f}{g(fh)^{1/2}} \left(\frac{f'}{f} + 2\frac{g'}{g} + 3\frac{h'}{h} \right) \to -8\frac{kM_g}{c^2} \frac{1+\beta^2-\beta}{1-\beta^2}.$$

The left hand side of (2.29) is continuous, i.e. this equation gives

$$-8\frac{kM_g}{c^2} \frac{1+\beta^2-\beta}{1-\beta^2} = 8\kappa \int_0^{r_0} r^2 \left(-T^{\overset{G}{4}}{}_4 \right) dr - 24\frac{kP}{c^2} \geq -24\frac{kP}{c^2}.$$

Hence, we get by virtue of (2.19)

$$M_g \left(1+\beta^2-\beta\right) \leq 3P\beta(1-2\beta). \tag{2.50}$$

The assumption $P=0$ implies by virtue of (2.48a) that the mass $M_g = 0$. Therefore, we have $P > 0$. Relation (2.50) can be rewritten

$$(M-P)\left(1+\beta^2-\beta\right) \leq -P\left(1-4\beta+7\beta^2\right) < 0,$$

i.e. we obtain

$$M < P. \tag{2.51}$$

An equation of state with velocity of sound c_s has the form

$$p = c_s{}^2 \rho, \quad c_s{}^2 \leq 1.$$

Hence, we get by integration the inequality

$$P \leq M$$

which is in contradiction to (2.51).

Therefore, every static spherically symmetric star has a radius $r_0 > K/\xi_c$, i.e. static spherically symmetric bodies have no singular solutions.

In empty space a singularity at a Euclidean distance from the centre can exist.

The radius of this singular sphere is smaller than the radius of the body. Hence, there is no event horizon, i.e. static black holes do not exist. Escape of energy and information is possible, i.e. no contradiction to quantum mechanics (see [Pet 14b]). It is worth to mention that the singularity -if it exists- is at a Euclidean distance and is not a singularity of the coordinate system as by general relativity.

2.5 Equations of Motion

In this sub-chapter the equations of motion of a test particle in a spherically symmetric gravitational field are studied.

Let us assume that the particle is moving in the plane given by the coordinates x^1 and x^2, i.e. $\vartheta = \pi/2$. The velocity is given in spherical polar coordinates by

$$\left(\frac{dr}{dt}, 0, \frac{d\phi}{dt}\right). \tag{2.52}$$

The equations (1.30) for a test particle can be written by the use of (2.4b)

$$\frac{d}{dt}\left(\frac{1}{f}\frac{dr}{dt}\frac{dt}{d\tau}\right) = \frac{1}{2}\left(-\frac{f'}{f^2}\left(\frac{dr}{dt}\right)^2 + \frac{r}{g}\left(2 - r\frac{g'}{g}\right)\left(\frac{d\varphi}{dt}\right)^2 + \frac{h'}{h^2}c^2\right)\frac{dt}{d\tau} \tag{2.53a}$$

$$\frac{d}{dt}\left(\frac{r^2}{g}\frac{d\varphi}{dt}\frac{dt}{d\tau}\right) = 0 \tag{2.53b}$$

$$\frac{d}{dt}\left(\frac{1}{h}\frac{dt}{d\tau}\right) = 0\,. \tag{2.53c}$$

The relation (1.13) has the form

$$c^2\left(\frac{d\tau}{dt}\right)^2 = \frac{c^2}{h} - \frac{1}{f}\left(\frac{dr}{dt}\right)^2 - \frac{r^2}{g}\left(\frac{d\varphi}{dt}\right)^2. \tag{2.54}$$

Equation (2.53c) yields

$$\frac{dt}{d\tau} = \alpha h \tag{2.55}$$

where α is a constant of integration. Equation (2.53b) implies with a further constant of integration

$$r^2 \frac{d\varphi}{dt}\frac{dt}{d\tau} = \beta g\ . \tag{2.56}$$

The last two relations give

$$r^2 \frac{d\varphi}{dt} = \frac{\beta}{\alpha}\frac{g}{h} \tag{2.57}$$

The equations (2.55) and (2.54) yield

$$\left(1-\frac{1}{\alpha^2 h}\right)\frac{c^2}{h} = \frac{1}{f}\left(\frac{dr}{dt}\right)^2 + \frac{r^2}{g}\left(\frac{d\varphi}{dt}\right)^2 . \tag{2.58}$$

Relation (2.57) corresponds to the second Kepler law. The equations (2.58) can be written

$$\left(\frac{dr}{dt}\right)^2 + r^2\frac{f}{g}\left(\frac{d\varphi}{dt}\right)^2 = c^2\left(1-\frac{1}{\alpha^2 h}\right)\frac{f}{h}\ . \tag{2.59}$$

Inserting (2.57) into equation (2.59) we get

$$\left(\frac{dr}{dt}\right)^2 = -\frac{1}{r^2}\left(\frac{\beta}{\alpha}\right)^2\frac{fg}{h^2} + c^2\left(1-\frac{1}{\alpha^2 h}\right)\frac{f}{h}\ . \tag{2.60}$$

The equation (2.60) is a differential equation of first order for $r(t)$. Knowing the solution of (2.60) we have a first order differential equation (2.57) for calculating $\varphi(t)$. These two functions describe the motion of the test particle in the spherically symmetric gravitational field. We will now give the differential equation which describes the trajectory of the test body. We eliminate the time t in the equations (2.57) and (2.59). Furthermore, we put

$$\rho = 1/r\ . \tag{2.61}$$

It follows

$$\frac{d\varphi}{dt} = \frac{\beta}{\alpha}\rho^2 \frac{g}{h}$$

and

$$\left(\frac{d\rho}{dt}\right)^2 = \rho^4\left(-\left(\frac{\beta}{\alpha}\right)^2 \rho^2 \frac{fg}{h^2} + c^2\left(1-\frac{1}{\alpha^2 h}\right)\frac{f}{h}\right).$$

The last two equations give

$$\left(\frac{d\rho}{d\varphi}\right)^2 = -\rho^2 \frac{f}{g} + c^2\left(\frac{\alpha}{\beta}\right)^2\left(h-\frac{1}{\alpha^2}\right)\frac{f}{g^2}\,. \tag{2.62}$$

The differential equation (2.62) describes the inverse ρ of the distance r as a function of the angle φ.

2.6 Redshift

In this sub-chapter the redshift of spectral lines in the gravitational field is studied. It follows by virtue of (1.8) for an atom at rest in the gravitational field the following relation between proper -time and absolute time

$$d\tau = \left(-g_{44}\right)^{1/2} dt = dt \,/\, \left(h(r)\right)^{1/2} \tag{2.63}$$

where (2.4b) is used. This relation gives for the frequency $\nu_e(r)$ of light emitted from an atom in the gravitational field at distance r from the centre of the body

$$\nu_e(r) = \nu_0 \,/\, \left(h(r)\right)^{1/2} \tag{2.64a}$$

where ν_0 is the frequency of light emitted from the same atom at infinity, i.e. neglecting gravitation. By virtue of Planck's law $E = h\nu$ where h is the Planck constant we get for the emitted energy

$$E(r) = E_0 \,/\, \left(h(r)\right)^{1/2}. \tag{2.64b}$$

This relation follows also by the definition of the energy

$$E \hat{=} -g_{4k}\frac{dx^k}{d\tau} \tag{2.65}$$

and the use of (2.4b) and (2.63). Let us assume that the atom at distance r_1 emits light which moves in the gravitational field to the distance r_2. By virtue of (1.30) the energy of light is not changing in the stationary, gravitational field, i.e. the energy (resp. frequency) of light received at r_2 is

$$\nu_r(r_1) = \nu_0 / \left(h(r_1)\right)^{1/2}. \tag{2.66}$$

Light emitted from the same atom at distance r_2 has the frequency

$$\nu_e(r_2) = \nu_0 / \left(h(r_2)\right)^{1/2}. \tag{2.67}$$

Hence the last two relations imply

$$\nu_e(r_2) / \nu_r(r_1) = \left(h(r_1) / h(r_2)\right)^{1/2}. \tag{2.68}$$

The redshift z is then given by

$$z = \frac{\lambda_r}{\lambda_e} - 1 = \frac{\nu_e(r_2)}{\nu_r(r_1)} - 1 = \left(\frac{h(r_1)}{h(r_2)}\right)^{1/2} - 1. \tag{2.69}$$

By virtue of (2.39c) we get to first order approximation

$$z \approx \frac{K}{r_1} - \frac{K}{r_2} \approx \frac{kM_g}{c^2}\left(\frac{1}{r_1} - \frac{1}{r_2}\right), \tag{2.70}$$

i.e. light emitted at r_1 and received at $r_2 > r_1$ gives a redshift z stated by (2.70) to the first order accuracy in agreement with the result of general relativity.

The result (2.70) is by the authors of article [*Pau* 65] experimentally verified in the gravitational field of the Earth with an altitude of $20m$ by the use of the Mössbauer-effect to an accuracy of 1% .

2.7 Deflection of Light

We consider a light ray coming from (r_1,φ_1), passing the nearest point $(r_0,0)$ to the centre of the body and then moving to the observer at (r_2,φ_2). The equations which describe the motion of this light ray are given in the sub-chapter 2.6. We start from the differential equation (2.62). For the nearest distance r_0 of the light ray to the centre of the body we have

$$\left(\frac{d\rho}{d\varphi}\right)_{\varphi=0}=0$$

implying by the use of (2.62) and (2.39) to first order approximation in K

$$\left(\frac{\alpha}{\beta}\right)^2\approx\frac{1}{\left(r_0c^2\right)}\left(1-4\frac{K}{r_0}+\frac{1}{\alpha^2}\right). \tag{2.71a}$$

Furthermore, we get for a light ray $d\tau=0$ by virtue of (2.55)

$$\frac{1}{\alpha}=0\,. \tag{2.71b}$$

Substituting the last two relations into equation (2.62) we receive to the first order approximation

$$\left(\frac{d\rho}{d\varphi}\right)^2=-\rho^2+\frac{1}{r_0^{\,2}}\left(1-4\frac{K}{r_0}+4K\rho\right). \tag{2.72}$$

The solution of this differential equation with the initial condition $\rho(0)=\rho_0=1/r_0$ can be given analytically. We have

$$\varphi=-\int_{\rho_0}^{\rho}\left(-\rho^2+4\frac{K}{r_0^{\,2}}\rho+\frac{1}{r_0^{\,2}}\left(1-4\frac{K}{r_0}\right)\right)^{-1/2}d\rho$$

Elementary integration and (2.61) give

$$r=r_0\,/\left(2\frac{K}{r_0}+\left(1-2\frac{K}{r_0}\right)\cos\varphi\right). \tag{2.73}$$

Inserting the starting point and the end point of the light ray we have for $i=1,2$

$$r_i = r_0 \Big/ \left(2\frac{K}{r_0} + \left(1 - 2\frac{K}{r_0} \right) \cos\varphi_i \right). \tag{2.74}$$

Put for $i=1,2$

$$\varphi_i = \pm\left(\frac{\pi}{2} + \psi_i \right) \tag{2.75a}$$

where the upper (lower) sign stands for $i=1\,(i=2)$ then we get from (2.74) to first order in K

$$\psi_i \approx 2\frac{K}{r_0} - \frac{r_0}{r_i}. \tag{2.75b}$$

Let γ_i be the angle between the tangent at the light curve in the point (r_i, φ_i) and the x^1 -axis we have

$$ctg\gamma_i = \left(\frac{1}{r_i}\frac{dr_i}{d\varphi_i}\cos(\varphi_i) - \sin\varphi_i \right) \Big/ \left(\frac{1}{r_i}\frac{dr_i}{d\varphi_i}\sin\varphi_i + \cos\varphi_i \right).$$

We have by virtue of (2.71) with (2.61)

$$\left(\frac{dr_i}{d\varphi_i} \right)^2 = r_i^2 \left(-1 + \left(\frac{r_i}{r_0} \right)^2 \left(1 - 4\frac{K}{r_0} + 4\frac{K}{r_i} \right) \right).$$

The last two relations together with (2.75) imply by elementary computations

$$ctg\gamma_i \approx \mp 2\frac{K}{r_0}.$$

The deflection of light is given by $\Delta\gamma = \gamma_1 - \gamma_2$. Hence, we have

$$\begin{aligned}\Delta\gamma &\approx tg(\Delta\gamma) = (tg\gamma_1 - tg\gamma_2)/(1+tg\gamma_1 tg\gamma_2)\\ &\approx ctg\gamma_2 - ctg\gamma_1\\ &\approx 4\frac{K}{r_0}.\end{aligned} \tag{2.76}$$

The formula (2.76) gives the deflection of light and it is identical with the result of general relativity to the studied approximation.

2.8 Perihelion Shift

We consider now a test particle in the orbit of a spherically symmetric body with velocity

$$|v|^2 = \left(\frac{dr}{dt}\right)^2 + \left(r\frac{d\varphi}{dt}\right)^2 \ll c^2.$$

Hence, we get from (2.58) and (2.39) to first order approximation to the accuracy of $O\left(\frac{1}{c^2}\right)$:

$$\frac{1}{\alpha^2} \approx 1 + 2\frac{K}{r} - \left(\frac{|v|}{c}\right)^2 \approx 1 - 2\frac{E}{M_g c^2}. \tag{2.77}$$

Here, the conservation law of energy of the test particle in the gravitational field is used to Newtonian accuracy and E is the classical energy satisfying

$$E \ll M_g c^2. \tag{2.78}$$

We get from (2.62) by the use of (2.77), (2.78) and (2.39) to second order in K

$$\left(\frac{d\rho}{d\varphi}\right)^2 = -\rho^2 + c^2\left(\frac{\alpha}{\beta}\right)^2\left(\frac{E}{M_g c^2} + 2K\rho + 6(K\rho)^2\right). \tag{2.79}$$

Put

$$\alpha_1 = 1 - 6(Kc)^2\left(\frac{\alpha}{\beta}\right)^2,\ \alpha_2 = Kc^2\left(\frac{\alpha}{\beta}\right)^2/\alpha_1, \alpha_3 = \frac{E}{M_g}\left(\frac{\alpha}{\beta}\right)^2/\alpha_1. \tag{2.80}$$

The analytic solution of (2.79) with the initial condition $\rho(\varphi_0) = \rho_0$ has the form

$$\rho = \alpha_2 + (\alpha_2^2 + \alpha_3)^{1/2} sin\left\{\alpha_1^{1/2}(\varphi - \varphi_0) + arcsin\left(\frac{\rho_0 - \alpha_2}{(\alpha_2^2+\alpha_3)^{1/2}}\right)\right\}. \quad (2.81)$$

The solution is an elliptic curve, i.e., there exist two values $\rho_1 > \rho_2 > 0$ such that the right hand side of (2.79). Hence,

$$(\rho_1 - \rho)(\rho - \rho_2) = -\rho^2 + 2\alpha_2\rho + \alpha_3 = 0.$$

This yields

$$\rho_1 + \rho_2 = 2\alpha_2, \ \rho_1\rho_2 = \alpha_3. \quad (2.82)$$

Equation (2.81) gives for a full period the angle

$$\Delta\varphi = \frac{2\pi}{\alpha_1^{1/2}} \approx 2\pi\left(1 + 3K^2c^2\left(\frac{\alpha}{\beta}\right)^2\right).$$

Therefore, we get a perihelion shift

$$\Delta\psi = \Delta\varphi - 2\pi = 6\pi K^2c^2\left(\frac{\alpha}{\beta}\right)^2 \quad (2.83)$$

in the direction of the motion of the test particle.

An elliptic curve with the semi-major axis $a$and the eccentricity $e$satisfies

$$\frac{1}{\rho_1} = a(1-e), \ \frac{1}{\rho_2} = a(1+e).$$

It follows by (2.82)

$$\frac{2}{a(1-e^2)} = \frac{1}{a(1-e)} + \frac{1}{a(1+e)} = \rho_1 + \rho_2 = 2\alpha_2 \approx 2Kc^2\left(\frac{\alpha}{\beta}\right)^2.$$

Inserting this relation into (2.83) we have

$$\psi = 6\pi\frac{kM_g}{c^2a(1-e^2)}. \quad (2.84)$$

Hence, we get for the perihelion shift of a test particle in a spherically symmetric gravitational field the same result as by Einstein's general theory of relativity.

2.9 Radar Time Delay

We consider a light ray starting from an observer at (r_2, φ_2), passing the spherically symmetric body at $(r_0, 0)$ and reflected at a body with coordinates (r_1, φ_1) and then travelling back to the observer on the same way. We will calculate the needed time and compare it with time when there is no gravitational field.

We start from equation (2.60) for a light ray, i.e., the relations (2.71) hold. Hence, it follows to first order in K

$$\left(\frac{dr}{dt}\right)^2 = -\left(\frac{r_0}{r}\right)^2 c^2 \left(1 + 4\frac{K}{r_0}\right)\frac{fg}{h^2} + c^2\frac{f}{h}.$$

Inserting (2.39) we get

$$\left(\frac{dr}{dt}\right)^2 = -\left(\frac{r_0}{r}\right)^2 c^2 \left(1 + 4\frac{K}{r_0}\right)/\left(1 + 8\frac{K}{r}\right) + c^2\left(1 - 4\frac{K}{r}\right).$$

Therefore, the time for the propagation of a radio signal from $(r_0, 0)$ to (r_i, φ_i) is

$$t(r_0, r_i) \approx \frac{1}{c}\int_{r_0}^{r_i} r\left(1 + 4\frac{K}{r}\right)/S(r)dr$$

where

$$S(r) = \left(r^2\left(1 + 4\frac{K}{r}\right) - r_0^2\left(1 + 4\frac{K}{r_0}\right)\right)^{1/2}.$$

Elementary integration gives

$$t(r_0, r_i) = \frac{1}{c}\left\{S(r_i) + 2K\, ln\left(\left(r_i + 2K + S(r_i)\right)/(r_0 + 2K)\right)\right\}.$$

We get to first order in K

$$\begin{aligned} t(r_0, r_i) &= \frac{1}{c}\left\{\left(r_i^2 - r_0^2\right)^{1/2} + 2K\left(\frac{r_i - r_0}{r_i + r_0}\right)^{1/2}\right. \\ &\quad \left. + 2K\, ln\left(\left(r_i + \left(r_i^2 - r_0^2\right)^{1/2}\right)/r_0\right)\right\} \\ &\approx \frac{1}{c}\left(r_i - \frac{1}{2}\frac{r_0^2}{r_i} + 2K + 2K\, ln\frac{2r_i}{r_0}\right). \end{aligned}$$

The time of propagation from (r_1, φ_1) to (r_2, φ_2) is

$$t(r_1, r_2) = t(r_0, r_1) + t(r_0, r_2)$$
$$\approx \frac{1}{c}\left(r_1 + r_2 - \frac{1}{2}\left(\frac{r_0^2}{r_1} - \frac{r_0^2}{r_2}\right) + 2K\left(2 + ln\frac{4r_1r_2}{r_0^2}\right)\right). \tag{2.86}$$

The Euclidean distance between $(r_1, \varphi_1)$and (r_2, φ_2) is

$$R = \{(r_1 cos\varphi_1 - r_2 cos\varphi_2)^2 + (r_1 sin\varphi_1 - r_2 sin\varphi_2)^2\}^{1/2}$$
$$\approx r_1 + r_2 - \frac{1}{2}\frac{r_1 r_2}{r_1 + r_2}(\psi_1 + \psi_2)^2$$

where (2.75a) is used. Inserting (2.75b) it follows to first order in K

$$R \approx r_1 + r_2 - \frac{1}{2}\frac{r_0^2}{r_1 + r_2}\left(\frac{r_2}{r_1} + \frac{r_1}{r_2}\right) - \frac{r_0^2}{r_1 + r_2} + 4K.$$

Hence, we get for the time delay Δt of the radio signal from (r_1, φ_1) to (r_2, φ_2) and back

$$\Delta t = 2\left(t(r_1, r_2) - \frac{R}{c}\right) \approx 4K\, ln\frac{r_1 r_2}{r_0^2}. \tag{2.87}$$

Formula (2.87) is identical with the corresponding result of general relativity in the case when harmonic coordinates are used whereas when Schwarzschild coordinates are considered additional expressions appear (see e.g. [*Wei* 72], [*Log* 86]) . In the theory of gravitation in flat space-time the distance is always the Euclidean one whereas in Einstein's general theory of relativity we have a non-Euclidean geometry implying the mentioned difficulty. Experimental results confirm the result (2.87) to high accuracy (see e.g. [*Sha* 71]).

These results about static spherically symmetric stars with the aid of theory of gravitation in flat space-time can be found in the articles [*Pet* 82, *Pet* 88].

Summarizing, the results of flat space-time theory of gravitation for static spherically symmetric stars agree with the corresponding ones of general relativity to high accuracy by virtue of weak gravitational fields.

2.10 Neutron Stars

To calculate neutron stars we have to solve the differential equations (2.10) and (2.12) together with an equation of state (2.13). The boundary conditions

are given by (2.14). The boundary r_0 of the neutron star follows from (2.15) and the mass is given by (2.19) together with (2.16). This problem seems to be not solvable analytically. Numerical methods must be used. The details of the numerical computations can be found in the paper [Sta 84]and only the results will be given. Several equations of state are considered. For $\rho(r) \leq 5 \cdot 10^{14}\ g/cm^3$ the table of [Bay 71] is used and then for $\rho(r) > 5 \cdot 10^{14}\ g/cm^3$ the equations of state are continued by the tables of several authors The results of the flat space-time theory of gravitation are given in the following tables where the author of the continued table is stated.

Table 1. [Bet 74]

$\rho(0) \cdot 10^{-15}$ *Geben Sie hier eine Formel ein.* g/cm ^{3}	$p(0) \cdot 10^{-1}$ g/cm ^{3}	$f(r_0)$	$g(r_0)$	$h(r_0)$	$M_g/M_\odot$	r_0 km
0.859	0.085	0.765	0.772	1.32	1.05	10.62
2.010	0.547	0.546	0.573	1.90	2.35	10.33
3.160	1.268	0.474	0.513	2.23	2.69	9.58
5.350	3.071	0.426	0.475	2.51	2.77	8.64

Hence, we get with the equation of state of [Bet 74] a maximal mass of $2.77 M_\odot$ with a radius of $8.64\ km$ and a central density of $5.350 \cdot 10^{15}\ g/cm^3$.

Table 2. [Wal 74]

$\varrho(0) \cdot 10^{-15}$ g/cm ^{3}	$p(0) \cdot 10^{-15}$ g/cm ^{3}	$f(r_0)$	$g(r_0)$	$h(r_0)$	$M_g/M_\odot$	r_0 km
1.149	0.315	0.526	0.553	1.99	2.88	12.03
2.132	0.974	4.425	0.469	2.54	3.61	11.33
3.060	1.651	0.405	0.455	2.69	3.64	10.82
4.547	2.785	0.399	0.452	2.73	3.53	10.36
8.360	5.829	0.401	0.455	2.69	3.40	10.04

This equation of state gives a maximal mass of a neutron star of $3.64 M_\odot$ with a radius of 10.82 km and a central density of matter of $3.06 \cdot 10^{15} \frac{g}{cm^3}$.

In the paper [Hae 81] several equations of state are studied and the maximal mass of neutron stars is calculated by the use of Einstein's general theory of relativity. Here, we will give for two equations of state the maximal mass, the radius and the density of matter in the centre of the star by flat space-time

theory of gravitation. In brackets the corresponding values of general relativity are stated.

$$M = 4.14M_{\odot}(= 2.5M_{\odot}),\ r_0 = 12.06\ km\ (= 12.1\ km),$$

$$\rho(0) = 2.664 \cdot 10^{15}\ g/cm^3\ (= 2.66 \cdot 10^{15}\ g/cm^3).$$

$$M = 5.13M_{\odot}(= 3.1M_{\odot}), r_0 = 14.83\ km\ (= 12.8\ km),$$

$$\rho(0) = 1.502 \cdot 10^{15}\ g/cm^3.$$

We see that although the radius of the neutron star has in both theories about the same value but the maximal mass can be greater in flat space-time theory of gravitation than that resulting by the use of general relativity.

At last we will calculate neutron stars with a stiff equation of state, i.e.

$$p = \rho - \rho_i + p_i \quad (i = 1,2)$$

where ρ_i and p_i are taken from the table [Bay 71] with

$$\rho_1 = 5.09 \cdot 10^{14}\ g/cm^3, \quad p_1 = 8.22 \cdot 10^{12}\ g/cm^3,$$

$$\rho_2 = 2.00 \cdot 10^{14}\ g/cm^3, \quad p_2 = 1.44 \cdot 10^{12}\ g/cm^3.$$

For $\rho(r) \leq \rho_i$ the equation of [Bay 71] is used again. We get the maximal mass, the approximate radius and the central density of matter

$$M_1 = 5.09 \cdot M_{\odot}, r_{10} = 13.42\ km,\ \ \rho_1(0) = 1.54 \cdot 10^{15}\ g/cm^3.$$

$$M_2 = 8.32 \cdot M_{\odot}, r_{20} = 21.06\ km, \rho_2(0) = 0.57 \cdot 10^{15}\ g/cm^3.$$

Again we remark that the maximal mass of a neutron star can be greater than that received by general relativity. The maximal mass of a neutron star calculated by Einstein's theory with a stiff equation of state is $3.2 \cdot M_{\odot}$[Rho 74].

Summarizing, the mass of any star estimated by observations may suggest a black hole for this star by general relativity whereas the star can be a neutron star by the use of gravitation in flat space-time.

Details about the numerical calculations and further results on neutron stars can be found in the paper [Sta 84]. Results on neutron stars based on Einstein's theory can be found e.g. in the books [Dem 85] and [Sha 83]. Static neutron stars which have the form of a geoid are numerically computed and can be found in [Neu 87] based on the theory of gravitation in flat space-time.