Chapter 8

Expanding or Non-Expanding Universe

8.1 Non-Expanding Universe

In this sub-chapter we will study non-expanding universes. It is well-known that the observed redshift of distant objects (galaxies, quasars) are interpreted as Doppler-effect, i.e. the observed universe is expanding. Furthermore, astrophysical observations indicate an accelerated expansion in the recent epoch.

In the previous chapter we have shown that an expanding universe can be received if the cosmological constant $\Lambda = 0$. This result can be used to explain the observed redshift of distant objects. If the cosmological constant $\Lambda \neq 0$ the third law of thermodynamics is violated. Hence, another form of the second law of Thermodynamics containing the whole energy of the universe is considered. This law implies that there are no expansion and no contraction of the universe and entropy is not produced in the course of time.

It is worth to mention that this law is also applicable in the special case $\Lambda = 0$.

8.2 Proper Time and Absolute Time

In addition to the system time t and the proper time τ in the previous chapters we define the absolute time t'.

The proper time $\tilde{\tau}$ for an object at rest is defined by

$$d\tilde{\tau} = \frac{1}{\sqrt{h(t)}} dt. \tag{8.1}$$

This gives for the whole proper time since the beginning of the universe

$$\tilde{\tau}(t) = \int_{-\infty}^{t} \frac{1}{\sqrt{h(t)}} dt. \tag{8.2}$$

The proper time is used in the study of general relativity.

In the beginning of the universe we have

$$a(t) \approx a(-\infty) > 0.$$

This implies by the use of (7.14b) that

$$h^{1/2}(t) \approx \frac{1}{\left(a(-\infty)\right)^3} \frac{2\kappa c^4 \lambda}{H_0^2} (H_0 t)^2 \tag{8.3}$$

for $t \to -\infty$. Hence, relation (8.2) implies the proper time $\tilde{\tau}(t)$ at any time t. The equation (7.18) can be written by the use of (7.14b)

$$\left(\frac{1}{a}\frac{da}{d\tilde{\tau}}\right)^2 = H_0^2\left(-\frac{\Omega_m K_0}{a^6} + \frac{\Omega_r}{a^4} + \frac{\Omega_m}{a^3} + \Omega_\Lambda\right). \tag{8.4}$$

The differential equation (8.4) is for $K_0 = 0$ identical with the equation given by general relativity. The case $K_0 = 0$ implies the singularity, i.e., the big bang by Einstein's theory. But K_0 must be greater than zero and must fulfil the condition (7.28) which avoids the singularity. Hence, relation (8.4) implies that for $a(t)$not too small that the result of flat space-time theory of gravitation agrees with that of general relativity, i.e. shortly after the big bang of Einstein's theory.

We will now introduce the absolute time t' by

$$dt' = \frac{1}{a(t)\sqrt{h(t)}}dt = \frac{1}{a(t)}d\tilde{\tau}. \tag{8.5}$$

This gives for the proper τ in the universe

$$(cd\tau)^2 = -a^2(t)\{|dx|^2 - (dct')^2\}. \tag{8.6}$$

Relation (8.6) implies for the absolute value of the light-velocity v_L:

$$|v_L| = \left|\frac{dx}{dt\prime}\right| = c. \tag{8.7}$$

Therefore, the absolute value of the light velocity in the universe is always the vacuum light velocity c. This is the reason that t' is denoted as absolute time. In the further study we will remark that the time t' has advantages relative to the use of the proper time $\tilde{\tau}$ although the proper time is measured by atomic clocks.

The equation (8.4) can be written by the use of (8.5) in the form

$$\left(\frac{da}{dt\prime}\right)^2 = \frac{H_0^2}{a^2}(-\Omega_m K_0 + \Omega_r a^2 + \Omega_m a^3 + \Omega_\Lambda a^6). \tag{8.8}$$

Furthermore, assume that a light ray is emitted at distance r at time t'_e resp. at time $t'_e + dt'_e$ and it is received by the observer at time $t' = 0$ resp. at time $0 + dt'$. Then, it holds by the use of (7.8)

$$r = c\int_{t\prime_e}^{0} dt' = -c\,t'_e,\ r = c\int_{t\prime_e + dt\prime_e}^{dt\prime} dt' = c(dt' - t'_e - dt'_e).$$

These two relations give

$$dt' = dt'_e.$$

This is a further reason that t' is the absolute time.

8.3 Redshift

We will now calculate the frequency emitted from a distant object at rest and received by the observer at rest at present time. The use of the absolute time t' simplifies the calculation although the use of the system time t and of the proper time $\tilde{\tau}$ would give the same result.

Let us assume that an atom at rest in a distant object emits a photon at time t'_e. The proper time is by virtue of (8.6):

$$d\tau = a(t'_e)dt'. \tag{8.9}$$

The energy of the emitted photon is

$$E \sim -g_{44}(t'_e)\frac{dt'}{d\tau} \sim a(t'_e)E_0. \tag{8.10}$$

The photon moves to the observer and it arrives at time $t' = t'_0$. Let (p_1, p_2, p_3, p_4) be the four-momentum of the photon in the universe with

$$p_4 = -E(t')/c.$$

Then, it follows from equation (1.30) with $i = 4$ by the use of (8.6) and (8.7)

$$\frac{d}{dt'}\left(g_{44}\frac{dt'}{d\tau}\right) = a(t')\frac{da}{dt'}(c^2 - |v_L|^2) = 0, \tag{8.11}$$

i.e., the energy of the emitted photon is constant during its motion. It is worth to mention that the conservation of the energy of the photon during its motion to the observer only holds by the use of the absolute time t'. Hence, we have by the law

$$E = h\nu$$

where here h denotes the Planck constant that the arriving photon has the frequency

$$\nu = a(t'_e)\nu_0. \tag{8.12}$$

Here, ν_0 is the frequency emitted by the same atom at rest and at present time. This gives the reshift

$$z = \frac{\nu_0}{\nu} - 1 = \frac{1}{a({t'}_e)} - 1. \tag{8.13}$$

This redshift formula is also received by the use of the proper time $\tilde{\tau}$ and the system time t. This results can be found in the article of Petry [Pet 08].

We will now give the distance-redshift relation. Equation (8.6) implies for light emitted at distance r at time ${t'}_e$ and received at $r = 0$ at time ${t'}_0$ by the use of (8.7)

$$r = c\int_{{t'}_e}^{{t'}_0} dt' = c({t'}_0 - {t'}_e). \tag{8.14}$$

Equation (8.8) yields by differentiation

$$2a\frac{da}{dt'}\left(-\frac{1}{a^2}\left(\frac{da}{dt'}\right)^2 + \frac{1}{a}\frac{d^2a}{dt'^2}\right) = H_0^2\left(4\frac{\Omega_m K_0}{a^5} - 2\frac{\Omega_r}{a^3} - \frac{\Omega_m}{a^2} + 2\Omega_\Lambda\right)\frac{da}{dt'}$$

This relation gives at present time ${t'}_0$:

$$\frac{d^2a({t'}_0)}{d{t'}_0^2} = H_0^2\left(1 + 2\Omega_m K_0 - \Omega_r - \frac{1}{2}\Omega_m + \Omega_\Lambda\right). \tag{8.15}$$

The redshift (8.13) is approximated by Taylor expansion and the use of (8.14)

$$z \approx H_0\frac{r}{c} + \left(1 - \frac{1}{2}\frac{1}{H_0^2}\frac{d^2a({t'}_0)}{d{t'}_0^2}\right)\left(H_0\frac{r}{c}\right)^2.$$

Hence, we get by (8.15) and (7.17) and neglecting small expressions the redshift:

$$z \approx H_0\frac{r}{c} + \frac{3}{4}\Omega_m\left(H_0\frac{r}{c}\right)^2. \tag{8.16}$$

We easily get the redshift formula to higher order by the use of Taylor expansion to higher order. By virtue of the use of the absolute time t' only differentiation to higher order of (8.8) are needed by virtue of (8.14).

For an expanding universe the redshift follows by the transformation

$$X^i = a(\tilde{\tau})x^i \quad \text{(i=1,2,3)} \tag{8.17a}$$

with the velocity

$$\frac{d}{d\tilde{\tau}}X^i = \frac{da(\tilde{\tau})}{d\tilde{\tau}}x^i \quad \text{(i=1,2,3).} \tag{8.17b}$$

The proper time τ of the universe with the coordinates (X^1, X^2, X^3) and the proper time $\tilde{\tau}$ is given by:

$$(cd\tau)^2 = -\sum_{k=1}^{3}(dX^k)^2 + \frac{2}{c}\sum_{k=1}^{3}\frac{1}{a}\frac{da}{d\tilde{\tau}}X^k dX^k d(c\tilde{\tau}) + (dc\tilde{\tau})^2\left(1-\left(\frac{1}{c}\frac{1}{a}\frac{da}{d\tilde{\tau}}|X|\right)^2\right). \tag{8.18}$$

Relation (8.18) implies that locally, i.e. $X^i = 0$ (i=1,2,3) the velocity of light is equal to the vacuumlight velocity. This result is connected with the ideas of Einstein that locally the pseudo-Euclidean geometry holds. We get by the substitution of (8.17) into relation (8.18)

$$d\tau = d\tilde{\tau},$$

i.e. any observer in the expanding universe is given by (8.18) and has the proper-time $\tilde{\tau}$. The theory of gravitation in flat space-time doesn't use (8.18) and it is therefore not further studied.

We will mention that a universe which at first contracts and then expands has no singularity, i.e. there is no big bang. Instead of the big bang we have a universe with a bounce.

In flat space-time theory of gravitation the redshift may be explained without expansion of space by the conservation of the whole energy (7.8) of the universe. It follows from (7.7) with (7.11) that the different kinds of matter, of radiation and of gravitation are transformed into one another in the course of time by the time-dependence of a and h. This is in analogy to the result that the gravitational field influences the redshift(see (2.69)).

There sults of this chapter can be found in several articles of Petry [*Pet* 97*b*, *98a, 98b, 02, 08, 11a, 13b*.

8.4 Age of the Universe

We will now calculate the age of the universe measured with absolute timet'. It follows by the use of (8.8) for the age after the minimum of the function$a(t')$ till the present time:

$$\begin{aligned} T_{t'}(t'_0) &= \int_{t'_1}^{t'_0} dt' = \int_{a_1}^{1} 1/\left(\frac{da}{dt'}\right) da \\ &= \frac{1}{H_0}\int_{a_1}^{1} \frac{a da}{(-\Omega_m K_0+\Omega_r a^2+\Omega_m a^3+\Omega_\Lambda a^6)^{1/2}} \\ &\geq \frac{1}{H_0}\int_{a_1}^{1} \frac{a da}{(-\Omega_m K_0+(\Omega_r+\Omega_m+\Omega_\Lambda)a^2)^{1/2}} \\ &= \frac{1}{H_0}\left(1-(-\Omega_m K_0+(1+\Omega_m K_0)a_1^2)^{1/2}\right) \approx \frac{1}{H_0}. \end{aligned} \tag{8.19}$$

Therefore, the age of the universe measured with absolute time is greater than $\frac{1}{H_0}$ independent of the density parameters, i.e. there is no age-problem. It seems that the use of the absolute time instead of the proper time is more natural. This is implied by the fact that the time difference at a distant object stated by two different events is measured by the observer at present time with the same value of time difference and the velocity of light is everywhere and at any time equal to the vacuum light velocity.

Summarizing: Flat space-time theory of gravitation gives cosmological models with bounce and without big bang. Furthermore, the models can be interpreted as non-expanding universe. The redshift is explained by the transformation of the different kindsof energy into one another in the course of time whereas the whole energy of the universe is conserved. This interpretation can also be found in the article of Petry [Pet 07]. It follows that the introduction of the absolute time t' simplifies the computations. The expansion of space was at earlier times the only interpretation of the redshift. In the meantime there are many authors who negate the expansion and assume that the redshift is intrinsic.