Boris V. Vasiliev

Superconductivity Superfluidity

Superconductivity and Superfluidity

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Annotation

1. The prevailing now BCS-theory is based on the phonon mechanism. The impetus for its development was the discovery of the isotope-effect in superconductors. As it has been observed, this effect (for some superconductors) has the same dependence on the mass of the isotope as the cut-off frequency of phonon spectrum.

It was found later that the zero-point oscillations of ions in the crystal lattice of metals are anharmonic. Therefore, the isotopic substitution leads to a change in the interatomic distances in the lattice, to a change in the density of the electron gas and hence to a change in the Fermi energy of the metal.

That was not known during the creation of the BCS-theory.

2. Conduction electrons in the metal are in a potential well. One must act photons or by heating, to pull them out from this well. The lowest level in the well is the level of zero-point oscillations. Quantum mechanics requires from particles at the lowest level to make these oscillations.

At a sufficiently low temperature, electrons form pairs. The mechanism of formation of these bosons does not matter. It is important that this mechanism began its work above the critical temperature of superconductor. At low temperature all bosons are collected at the lowest level and all have the same energy. That means that they have the same frequencies and amplitudes of zero-point oscillations.

But pairs of the same energy can have different phases and orientation (polarization) of the zero-point oscillations. If the temperature is not low enough (above the interaction energy of steam), the interaction between them can be neglected. At that the oscillating with different phases and polarizations pairs will be described by different wave functions. They are not identical.

3. Due to the fact that the charged particles oscillate, there is an interaction between them. At sufficiently low temperatures, this interaction leads to a kind of ordered structure of zero-point oscillations at which due to their mutual attraction their energy decreases. As a result, this attraction forms a superconducting condensate, which is not scattered by

the defects, if the scattering energy is less than the energy of attraction.

4. Calculations show that the ratio of the critical temperature of formation of such condensate to the metal Fermi temperature:

$$\frac{T_c}{T_F} \simeq 5\pi\alpha^3 \simeq 6 \cdot 10^{-6} \tag{1}$$

(where α is the fine structure constant).

The critical magnetic field in this model is a field that destroys the coherence of the zero-point oscillations of the particles of the condensate.

These results of calculations are in good agreement with measured data for all superconductors.

It is appropriate to emphasize that the BCS-theory there are no workable formula that determines the specific critical parameters of superconductors.

5. Zero-point oscillations of shells of neutral atoms in the s-state were considered by F.London (in 1937). He showed that the helium atoms arrange the oscillations of their shells at about 4K. This ordering is energetically favorable, since in this case the attraction arises between atoms. He paid no attention, that only one oscillation mode is ordered at this temperature. But this is enough for the helium liquefaction as a repulsion is absent in the gas of neutral bosons.

The complete arrangement appears at twice less temperature. The calculation shows that the temperature of complete ordering of zero-point oscillations depends on the universal constants only:

$$T_0 = \frac{1}{3}\frac{M_4 c^2 \alpha^6}{k} = 2.177K. \tag{2}$$

This value is a very good agreement with the measured value of the superfluid transition temperature $T_\lambda = 2.172K$. This difference in the 4 digits may occur because of the inaccuracy of the calibration of thermometers, which are very difficult in this temperature range.

Also in this case it is possible to calculate the density of the superfluid condensate in liquid helium. It turns out that the density of particles in the condensate, as well as T_0,

depends on the ratio universal constants only:

$$n_0 = \frac{\alpha^2}{a_B^3} \sqrt{\frac{M_4}{2m_e}} \cong 2.172 \cdot 10^{22} \ atom/cm^3. \tag{3}$$

(where a_B is the Bohr radius).

Since all helium atoms pass into condensate at a low temperature, then it is possible to calculate the density of the liquid helium:

$$\gamma_4 = n_0 M_4 \cong 0.1443 \ g/cm^3, \tag{4}$$

which agrees well with the measured density of liquid helium equal to $0.145 \ g/cm^3$.

It should be emphasized that the above formulas are derived from the consideration of the mechanism of zero-point oscillations ordering and they were not known previously.

6. Thus the consent of the above formulas with the measurement data clearly shows that both - superconductivity and superfluidity - are the result of the work of the same mechanism - both of these related phenomena occur as a result of ordering zero-point oscillations.

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Part I
Preface - On the Disservice of Theoretical Physics

Chapter 1

Is It Possible for a Physical Theory to be Harmful?

One should not think that the fundamental scientific knowledge can be harmful. Most of theoretical physicists adequately reflects the physical reality and forms the basis of our knowledge of nature. However, some physical theories arised in the twentieth century are not supported by experimental data. At the same time impression of their credibility masked by a very complex mathematical apparatus is so great that some of them are even awarded the Nobel Prize. However, the fact it does not change - a number of generally accepted theories created in the twentieth century are not supported by the experience and therefore should be recognized as pseudoscientific and harmful.

The twentieth century ended and removed with every year further and further from us. Already possible to summarize its scientific results. The past century has brought great discoveries in the field of physics. At the beginning of the XX century was born and then rapidly developing nuclear physics. It was probably the greatest discovery. It radically changed the whole material and moral character of the world civilization. In the early twentieth century, the radio was born, it gradually led to the television, radiotechnics gave birth to computers. Their appearance can be compared with the revolution that occurred when people have mastered fire. The development of quanta physics leds to the emergence

of quantum devices, including the lasers shine. There is a long list of physical knowledge, which gave us the twentieth century.

1.1   Experimentalists and Theoreticians

The important point is that the twentieth century has led to the division of physicists on experimentalists and theorists. It was a natural process caused by the increasing complexity of scientific instruments and mathematical methods for constructing theoretical models. The need for the use of the vacuum technics, the low-temperature devices, the radio-electronic amplifiers and other subtle techniques in experimental facilities has led to the fact that the experimenters could be the only people who can work not only with your head but can do something their own hands. On the contrary, people are more inclined to work with the mathematical formalism could hope for success in the construction of theoretical models. This led to the formation of two castes or even two breeds of people.

In only very rare cases physicists could be successful on the both experimental and theoretical "kitchen".

The most striking scientist of this type was Enrico Fermi. He was considered as their own in the both experimental and theoretical communities.

He made an enormous contribution to the development of quantum and statistical mechanics, nuclear physics, elementary particle physics, and at the same time created the world's first nuclear reactor, opening the way for the use of nuclear energy.

However, In most cases experimentalists and theorists is very jealous of each other. There are many legends about what theorist is sad sack. So there was a legend about the Nobel Prize winner - theorist Wolfgang Pauli, according to which there was even a special "Pauli effect", which destroyed the experimental setup only at his approach.

One of the most striking instances of this effect, according to legend, took place in the laboratory of J. Frank in Gottingen. Where a highly complex experimental apparatus for the study of atomic phenomena was destroid in a completely inexplicable reason. J. Frank wrote about the incident Pauli in Zurich. In response, he received a letter with a

Danish mark, in which Pauli wrote that he depart to see on the Niels Bohr, and during a mysterious accident in the J. Frank laboratory he was in the train which just made a stop in Gottingen.

At the same time, of course, theorists began to set the tone in physics, because they claimed they can understand all physics wholly and to explain all of its special cases.

Outstanding Soviet theorist of the first half of the twentieth century was Ya. Frenkel. He wrote a lot of very good books on various areas of physics. Even a some anecdote went about his ability to explain everything. Supposedly once some experimenter caught his at a corridor and showed a some experimentally obtained curve. After a moment of thinking, Ja. Frenkel gave an explanation of the curve course. That it was noted that the curve was accidentally turned upside down. After this rotating it in place and a little reflection, Ja. Frenkel was able to explain this dependence too.

1.2   On the Specifics of the Experimental and the Theoretical Working

The features of relations of theoreticians and experimentalists to their work are clearly visible on the results of their researches.

These results are summarized for illustrative purposes in Table 1.1.

The situation with experimental studies is simple.

At these studies, various parameters of samples or the properties of the physical processes are measured.

If such measurements are not supplemented by a theoretical description of the mechanisms that lead to these results, this study can be regarded as a purely experimental. It can be placing in the box 1 in the table.

If an experimental study is complemented by a description of the theoretical mechanism that explains the experimental data, it's just good physical research. Put such work in the box 2.

Also the different situation is possible when the theoretical study of the physical effect or object is brought to the "numbers" which is compared with the measured data. That is essentially to think, that these studies are of the same type as the studies in box 2. However, as there is an emphasis on the theory of physical phenomena, these studies can be placed in the box 3.

As a result of this classification, the theoretical studies which have not been confirmed experimentally must be placed in box 4.

A correct theory - a very powerful tool of cognition. It is often difficult to understand the intricacies of experimentalists settings and theorists calculations. In this case, a theory comes to rescue. If a some researcher, for example, in the study of electromagnetic phenomena argues that do not fit into the framework of Maxwell's theory, there is no need for a closer analysis of his reasoning. Somewhere this researcher makes error. The Maxwell's theory so thoroughly tested experimentally and confirmed by the work of the entire electrical and radio technology, it makes no sense to attach importance to the assumptions which are contrary to it.

However, this power and severity of narrowing extends sometimes to any known theory. One can attribute a series of theories to as a commonly accepted. This can be said for example about the BCS-theory of superconductivity or the quark theory of elementary particles. These theories received the full recognition and even the Nobel Prizes. And it can be perceived as a proof of their correctness. It seems that it can be perceived as a proof of their correctness. However, the situation with their experimental confirmation is worse.

The BCS-theory is quite successful in explaining some of the properties that are common for superconductors - the emergence of the energy gap and its temperature dependence, the characteristic behavior of the specific heat of superconductors, the isotope effect in a number of metals, etc. However, the main properties of specific superconductors - their critical parameters - the BCS-theory does not explain. In reviews on superconductivity (and on superfluidity) abound formulas describing generalized characteristics and properties, but they are almost never brought to the typical "number", which is known from measurements.

The quark-theory also has weaknesses in its proof. At the foundation of the modern

theory of quarks, the assumption has laid down that there are particles which charges are aliquot to 1/3 e. However, these particles were not detected. To explain this fact the additional assumptions should be taken. But it is important that the numerical values of the characteristic properties of one of the fundamental particles - the neutron - can only be explained by assuming that the neutron and proton have the same quark structure [15].

Surprisingly, there are quite a few of these theoretical compositions.

Despite the obvious speculative nature of such theories, some of them received full recognition in the physics community.

Naturally, the question arises how bad the theoretical approach which is used to describe these phenomena, because it violates the central tenet of natural science.

Table 1.1   *The systematics of physics research.*

1.	the experimental research
2.	the experimental research + theoretical explanation of its results = physics
3.	the theoretical mechanism + confirming its measurement data = physics
4.	the theoretical studies have not yet confirmed by the experimental data

1.3   The Central Principe of Science

The central principe of natural science was formulated more than 400 years ago by William Gilbert (1544-1603).

One might think that this idea, as they say, was in the air among the educated people of the time. But formulation of this postulate has come down to us due to W. Gilbert's book [1].

It formulated simply: "All theoretical ideas claiming to be scientific must be verified experimentally".

Until that time false scientific statements weren't afraid of an empirical testing. A flight of fancy was incomparably more refined than an ordinary and coarse material

world. The exact correspondence of a philosophical theory to a experiment was not necessary. That almost discredited the theory in the experts opinion. The discrepancy of a theory and observations was not confusing at that time. In the course there were absolutely fantastic ideas from our point of view. So W. Gilbert writes that he experimentally refuted the popular idea that the force of the magnet can be increased, rubbed with garlic. Moreover one of the most popular question at the religious and philosophical debates had the quantitative formulation: how many angels can stay on the tip of the needle?

Galileo Galilei (1564-1642) lived a little later W. Gilbert had developed this doctrine and formulated three phase of testing of theoretical propositions:

1. *to postulate a hypothesis about the nature of the phenomenon, which is free from logical contradictions;*

2. *on the base of this postulate, using the standard mathematical procedures, to conclude laws of the phenomenon;*

3. *by means of empirical method to ensure, that the nature obeys these laws (or not) in reality, and to confirm (or not) the basic hypothesis.*

The use of this method gives a possibility to reject false postulates and theories.

1.4 The Characteristic Properties of Pseudo-Theories of XX Century

In the twentieth century, there were several theories that do not satisfy to the general postulate of science.

Many of them simply are not brought to ensure that their results could be compared with the measurement data of the objects. Therefore it is impossible to assess their scientific significance.

These pseudo-theories use always complicated mathematical apparatus, which tends to replaces them the required experimental confirmation.

Simplistically the chain of reasoning, which can be formed, for example, by a student at his acquaintance with these theory may be as the next sequence:

• theory created by the author is very complex;

• this means that the author is very smart and knows a lot;

• so smart and well-trained theorist should not be mistaken;

• it means his theory is correct.

All links in this chain of reasoning may be correct. Except the last. Theory is valid only if it is confirmed by experiments.

It is essential that pseudo-theories can not be simplified for obtaining of an approximate, but correct and simple physical constructions.

The correct approach to the explanation of the object can be mathematically difficult, if it aimed on an accurate description of the properties of the object. This approach should allow to get a simple estimation on the order of value.

Another feature of pseudo-theories consists in substitution of experimental proofs. All objects under consideration of physical theories have main individual properties that can be called paramount. For stellar physics they are individual for each star radii, temperatures, masses. For superconductors - individual for each critical temperatures and magnetic fields, for superfluid helium - the transition temperature and the density of atoms near it, and so on.

Quasi-theories are not able to predict the individual paramount properties of considered objects. They replace the study of the physical mechanisms of the formation of these primary parameters on a describing of general characteristics of the physics of the phenomenon and some of its common properties. For example, the theory of XX-th century substituted the explanation of the properties of specific superconductors by the prediction of the observed temperature dependence of the critical field or the energy gap which are characteristic for this phenomenon. As a result, it appears that there is an agreement between theory and experiment, although the general characteristics of the phenomenon can usually be called thermodynamic.

Let consider some specific pseudo-theory by theoretical physics in the twentieth century.

Chapter 2

About Pseudo-Theories of XX Century

2.1 The Theory of the Internal Structure of Hot Stars

Some theoretical constructs could be built only speculatively, since desired experimental data was not existed.

Astrophysics until the end of the twentieth century were forced to create a theory of the internal structure of stars, relying on the knowledge of "earthly" laws and intuition.

The modern astrophysics continues to use the speculative approach. It elaborates qualitative theories of stars that are not pursued to such quantitative estimates, which could be compared with the data of astronomers [5], [6].

The technical progress of astronomical measurements in the last decade has revealed the existence of different relationships that associate together the physical parameters of the stars.

log T/T$_o$
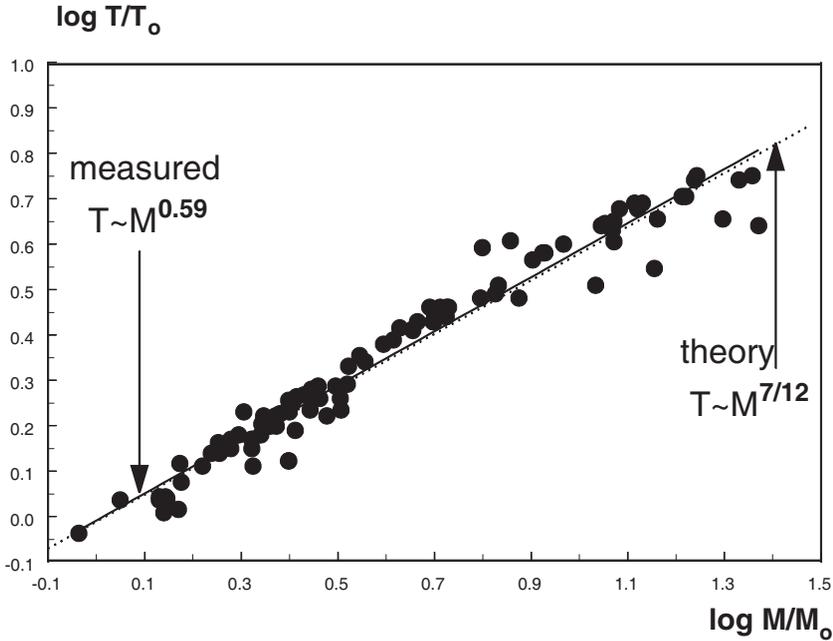
Figure 2.1 *Theoretical dependence of the surface temperature on the mass of the star in comparison with the measurement data. The theory takes into account the presence of the gravity induced electric polarization of stellar plasma. Temperatures are normalized to the surface temperature of the Sun (5875 K), the mass - to the mass of the Sun [7].*

To date, these dependencies are already accumulated about a dozen. The temperature-radius-mass-luminosity relation for close binary stars, the spectra of seismic oscillations of the Sun, distribution of stars on their masses, magnetic fields of stars (and etc.) have been detected. All these relationships are defined by phenomena occurring inside stars. Therefore, a theory of the internal structure of stars should be based on these quantitative data as on boundary conditions.

Of course, the astrophysical community knows about the existence of dependencies of stellar parameters which was measured by astronomers. However, in modern astrophysics it is accepted to think, that if an explanation of a dependency is not found, that it can be referred to the category of empirical one and it need no an explanation.

It seems obvious that the main task of modern astrophysics is the construction of a

theory that can explain the regularity of parameters of the Sun and stars which was detected by astronomers.
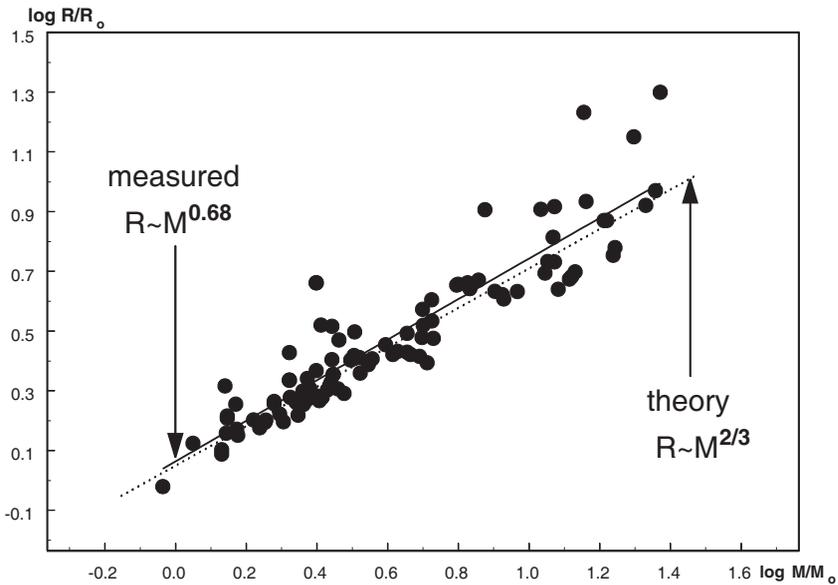
Figure 2.2 *Theoretical dependence of the radius of the star on its mass in comparison with the measurement data. The theory takes into account the presence of the gravity induced electric polarization of stellar plasma. Radius expressed in units of the solar radius, mass - in units of mass of the Sun [7].*

The reason that prevents to explain these relationships is due to the wrong choice of the basic postulates of modern astrophysics. Despite of the fact that all modern astrophysics believe that the stars consist from a plasma, it historically turned out that the theory of stellar interiors does not take into account the electric polarization of the plasma, which must occur within stars under the influence of their gravitational field. Modern astrophysics believes that the gravity-induced electric polarization (GIEP) of stellar plasma is small and it should not be taken into account in the calculations, as this polarization was not taken into account in the calculations at an early stage of development of astrophysics, when about a plasma structure of stars was not known. However, plasma is an electrically polarized substance, and an exclusion of the GIEP effect from the calculation is unwarranted. Moreover without of the taking into account of the GIEP-effect, the equilibrium stellar matter can not be correctly founded and a

theory would not be able to explain the astronomical measurements. Accounting GIEP gives the theoretical explanation for the all observed dependence [7].
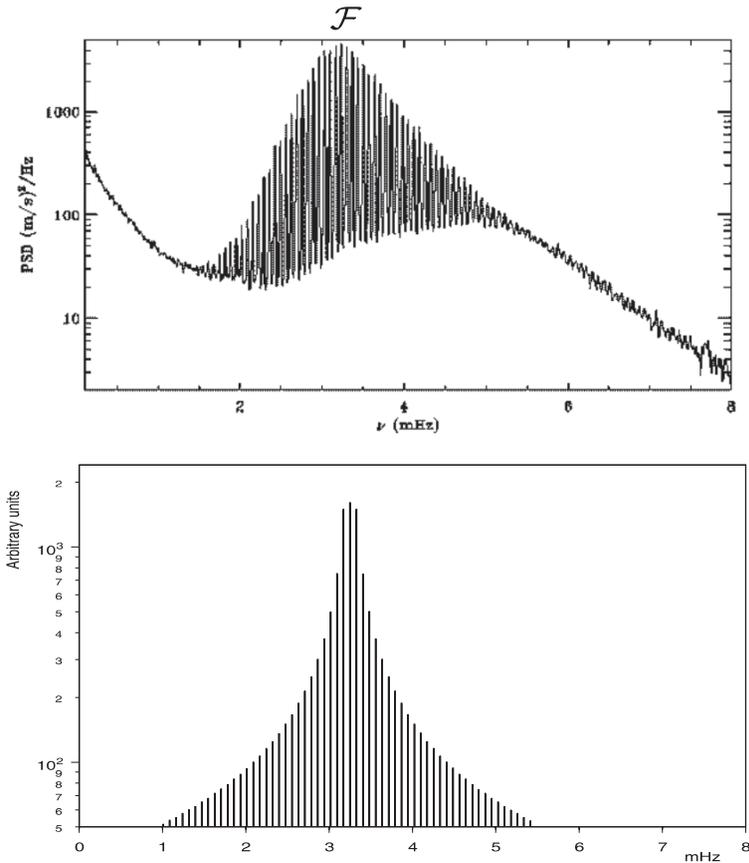
Figure 2.3 (a) - *The measured power spectrum of solar oscillation. The data were obtained from the SOHO/GOLF measurement [8]. (b) - The theoretical spectrum calculated with taking into account the existence of electric polarization induced by gravity in the plasma of the Sun [7].*

So the figures show the comparison of the measured dependencies of the stellar radius and the surface temperature from the mass of stars (expressed in solar units) with the results of model calculations, which takes into account the effect GIEP (Figure 2.1, 2.2).

The calculations with accounting of the GIEP-effect are able to explain the observed

spectrum of seismic solar oscillations (Figure 2.3) and measurements of the magnetic moments of all objects in the solar system, as well as a number of stars (Figure 2.4).
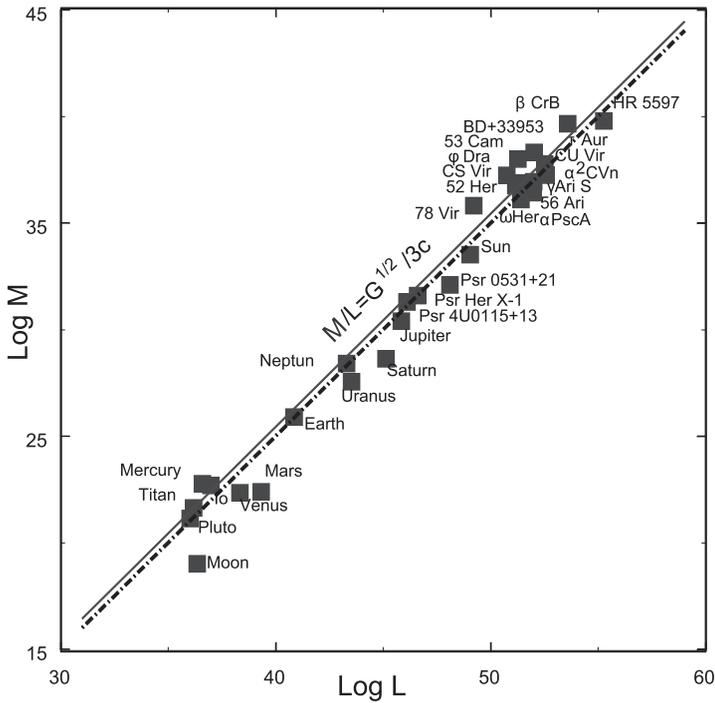
Figure 2.4   *The observed magnetic moments of cosmic bodies vs. their angular momenta [11]. On the ordinate: the logarithm of the magnetic moment over $Gs \cdot cm^3$. On the abscissa: the logarithm of the angular momentum over $erg \cdot s$. The solid line is according to Blackett's dependence [10].*

In general, the accounting of GIEP effects gives the explanation to all the data of astronomical measurements by building the star theory, in which the radius, mass, and temperature are expressed by the corresponding ratios of the fundamental constants, and individuality of stars are determined by two parameters - by the charge and mass numbers of nuclei, from which a stellar plasma is composed.

The important feature of this stellar theory, which is built with the GIEP acconting, is the lack of a collapse in the final stage of the star development, as well as "black holes" that could be results from a such collapse.

Only by relying on measurement data, physics of stars can get rid of speculations and obtain a solid foundation on which must be built any physical science.

2.2   The Theory of Terrestrial Magnetic Field

The modern theory of terrestrial magnetism trys to explain why the main magnetic field of the Earth near the poles is of the order 1 Oe.

According to the existing theoretical solution of this problem, there is a special mechanism of hydro-dynamo which generates electric currents in the region of the Earth's core [9]. this model was developed in the 1940's-1950's. At present it is generally adopted. Its main task - to give an answer: why the main magnetic field of the Earth near the poles is of the order of 1 Oe?

Such statement of the basic problem of terrestrial magnetism models nowadays is unacceptable. Space flights, started in 1960's, and the further development of astronomy have allowed scientists to obtain data on magnetic fields of all planets of Solar system, as well as some their satellites and a number of stars. As a result, a remarkable and earlier unknown fact has been discovered. It appears that the magnetic moments of all space bodies (those which have been measured) are proportional to their angular momenta. The proportionality coefficient is approximately equal to $G^{1/2}/c$, where G is the gravitational constant, c is the speed of light. See Figure 2.4.

Amazing is that this dependence remains linear within 20 orders of magnitude! This fact makes it necessary to reformulate the main task of the model of terrestrial magnetism. It should explain, first, why the magnetic moment of the Earth, as well as of other space bodies, is proportional to its angular momentum and, second, why the proportionality coefficient is close to the above given ratio of world constants.

As the pressure in the Earth's core is large enough to break the outer electron shells of atomic substances, this core should consist of an electron-ion plasma. The action of gravity on such a plasma lead to its electric polarization into the Earth core. The rotation of electrically polarized core (along with the entire planet) induces the terrestrial magnetic moment.

The magnetic moment and the moment of the rotation of Earth can be calculated in the framework of the model of the Earth at a minimizing its total energy The results of these calculations are in good agreement with measured data cite BV-terra.

This mechanism, which is a consequence of the law of universal gravitation, is workable in the case of all other (large) celestial bodies.

2.3   The Physics of Metal - The Thermo-Magnetic Effect

Among the theories of the twentieth century, there is another, which is based on an erroneous understanding of the mechanism of the considered phenomenon.

The main subject of study of the physics of metals is the behavior of a gas of conduction electrons. The characteristic properties of metals - their high thermal and electrical conductivity - are due to the existence of free conduction electrons.

In considering the mechanism of heat conduction in metals, it is assumed that the heat transfer is carried out by flow of hot electrons moving from the heated area of a metal in the cold one.

This hot stream displaces the cold electrons, which are forced to flow in opposite direction.

Since we are considering a homogeneous metal, the theory of this phenomenon assumes that these counter-currents flow diffusely. A flow of two diffuse counter-currents of equal magnitude suggests a complete absence of induced magnetic fields.

This point of view on considered process established in the early twentieth century. On their basis, the theory of thermoelectric phenomena in metals was developed, which predicted full absence of thermo-magnetic effect in metals.

However, the thermo-magnetic effect in metals exists [14], it is quite large and it can be easy found with the help of modern magnetometer.

The theoretical mistake arose from the fact that even in a completely homogeneous metal sample the counter-currents repel each other.

As a result of the repulsion of opposite flows of hot and cold electrons in a metal arises their convection. It induces a magnetic field inside and in the vicinity of the sample.

The corrected theory takes into account the thermo-magnetic effect [14], fits well into the overall picture of thermal phenomena in metals.

2.4   Elementary Particle Physics

The basis of modern elementary particle physics is considered to be the quark model.

The formation of this theory seems quite natural in the chain of sciences on the structure of matter: all substances consist of atoms and molecules. The central element of atom is nucleus. Nucleus consists of protons and neutrons, which in turn are composed of quarks.

The quark model assumes that all elementary particles are composed of quarks. In order to describe all their diversity, the quarks must have a fractional electric charge (equal to 1/3 e or 2/3 e) and other discrete properties, referred to as flavor, color, etc.

In the 60 years after the formulation of the foundations of the quark model, many experimenters sought to find particles with fractional charge.

But to no avail.

After that was coined by the confinement, ie property of quarks, prohibiting them in any way to express themselves in a free state.

Once something like that happened in the history of European culture. To some extent, this situation is reminiscent of the medieval concept of angels. Nobody doubted in an existence of angels, but they were attributed a property of the full indetectability, i.e. a peculiar confinement.

In modern physics, there is a handy method when nonexistent in nature particles are entered for convenience of description of certain phenomenon. For example, the phonons in crystals well describe many phenomena, but they are only the best method for studying these phenomena. Phonons are quasi-particles, ie, they do not really exist, but they are successful and convenient theoretical abstraction.

If one treats the quarks also as quasi-particles, their existence does not require experimental evidence. At that the convenience and the accuracy of the description come to the fore for them.

Really, the quark model aptly describes some experiments on the scattering of particles at high energies, for example, the formation of jets or a feature of the high-energy particles scattering of without their breaking.

However, that is not very strong argument.

The basic quarks of the first generation (u and d) are introduced in such a way that their combinations could explain the charge parameters of protons and neutrons. Naturally, the neutron is considered at that as an elementary particle like the proton. In the 30s of the XX-th century, theoretical physicists have come to the conclusion that a neutron must be an elementary particle without relying on the measurement data, which was not at that time.

Are there currently required measurements? Yes. The neutron magnetic moment and the energy of its beta-decay were measured and they can be calculated based on some model.

Let us assume that a neutron is composed particle, and, as well as the Bohr's hydrogen atom, it consists of a proton and an electron, which rotates around proton on a very small distance from it. On a very small distance from the proton, the electron motion becomes relativistic.

Calculations show that the magnetic moment of such relativistic Bohr's atom depends on universal constants only, and therefore it can be calculated with very great accuracy. Using the standard formulas of electrodynamics (excluding any impact electro-weak interaction), we find that the magnetic moment of the relativistic hydrogen "atom" (in Bohr's nuclear moment units) is [15]:

$$\mu_n \approx -1.91352, \tag{2.1}$$

i.e. it very well agrees with the experimentally measured magnetic moment of the neutron:

$$\frac{\mu_n(calc)}{\mu_n(meas)} = \frac{-1.91352}{-1.91304} \approx 1.00025 \tag{2.2}$$

This coincidence confirms the assumption that the neutron is not an elementary particle.

Additionally, this conclusion is supported by other calculations.

If to determine the energy of interaction inside the such relativistic hydrogen atom, we can estimate the maximum kinetic energy that can be obtained by an electron at the β-decay of the relativistic hydrogen atom. This account of electromagnetic forces (without the involvement of the theory of electro-weak interactions) produces the result that coincides with the measured energy of the neutron β-decay within a couple of percent [15].

The agreement of this model with measured data suggests that the neutron is not an elementary particle, and therefore it can not be described by the theory of quark and quark model itself must be subject to audit.

2.5   Superconductivity and Superfluidity

These two super-phenomena were discovered in the early 20th century and for a long time remained the most mysterious in condensed matter physics. Consideration of these phenomena and the development of their theories are given in the following section of this book.

Part II

The Development of the Science of Superconductivity and Superfluidity

Chapter 3

Introduction

3.1 Superconductivity and Public

Superconductivity is a beautiful and unique natural phenomenon that was discovered in the early 20th century. Its unique nature comes from the fact that superconductivity is the result of quantum laws that act on a macroscopic ensemble of particles as a whole. The concept of superconductivity is attractive not only for circles of scholars, professionals and people interested in physics, but wide educated community.

Extraordinary public interest in this phenomenon was expressed to the scientific community just after the discovery of high temperature superconductors in 1986. Crowds of people in many countries gathered to listen to the news from scientific laboratories. This was the unique event at this time, when the scientific issue was the cause of such interest not only in narrow circle of professionals but also in the wide scientific community.

This interest was then followed by public recognition. One sign of this recognition is through the many awards of Nobel Prize in physics. This is one area of physical science, where plethoras of Nobel Prizes were awarded. The chronology of these awards follows:

1913: Heike Kamerlingh-Onnes was awarded the Nobel Prize in Physics for the

discovery of superconductivity.

1962: Lev Landau was awarded the Nobel Prize in Physics in for the pioneering theories for condensed matter, especially liquid helium, or in other words, for the explanation of the phenomenon of superfluidity.

1972: John Bardeen, Leon N. Cooper and J. Robert Schrieffer shared the Nobel Prize in Physics for the development of the theory of superconductivity, usually called the BCS theory.

1973: Brian D. Josephson was awarded the Nobel Prize in Physics in for the theoretical predictions of the properties of the superconducting current flowing through the tunnel barrier, in particular, the phenomena commonly known today under the name of the Josephson effects.

1978: Pyotr Kapitsa was awarded the Nobel Prize in Physics, for his basic inventions and discoveries in the area of low-temperature physics, that is, for the discovery of superfluidity.

1987: Georg Bednorz and Alex Muller received the Nobel Prize in Physics for an important breakthrough in the discovery of superconductivity in ceramic materials.

2003: Alexei Abrikosov, Vitaly Ginzburg and Anthony Legett received the Nobel Prize in Physics for pioneering contributions to the theory of superconductors and superfluids.

Of course, the general attention to superconductivity is caused not just by its unique scientific beauty, but in the hopes for the promise of huge technological advances. These technological advances pave the way, creating improved technological conditions for a wide range of applications of superconductivity in practical societal uses: maglev trains, lossless transmission lines, new accelarators, devices for medical diagnostics and devices based on highly sensitive SQUIDs.

Because of these discoveries, it may now look like there is no need for the development of the fundamental theory of superconductivity at all. It would seem that the most important discoveries in superconductivity have been already made, though more or less randomly.

Isaac Kikoine, a leading Soviet physicist, made a significant contribution to the study

of superconductivity on its early on stage.[1]

He used to say, whilst referring to superconductivity, that many great scientific discoveries was made by Columbus method. This was when, figuratively speaking, America was discovered by a researcher who was going to India. This was a way by which Kamerlingh-Onnes came to his discovery of superconductivity, as well as a number of other researchers in this field.

Our current understanding of superconductivity suggests that it is a specific physical discipline. It is the only area of physics where important physical quantities equals exactly to zero. In other areas of physics small and very small values exist, but there are none which are exactly zero. A property can be attributed to the zero value, in the sense there is a complete absence of the considered object. For example, one can speak about a zero neutrino mass, the zero electric charge of neutrons, etc., but these terms have a different meaning.

Is the electrical resistance of superconductors equal to zero?

To test this, H. Kamerlingh-Onnes froze a circulating current in a hollow superconducting cylinder. If the resistance was still there, the magnetic field this current generated would reduced. It was almost a hundred years ago when Kamerlingh Onnes even took this cylinder with the frozen current from Leiden to Cambridge to showcase his findings. No reduction of the field was found.

Now it is clear that resistance of a superconductor should be exactly equal to zero. This follows the fact that the current flow through the superconductor is based on a quantum effect. The behavior of electrons in a superconductor are therefore governed by the same laws as in an atom. Therefore, in this sense, the circulating current over a superconductor ring is analogous to the movement of electrons over their atomic orbits.

Now we know about superconductivity, more specifically ł it is a quantum phenomenon in a macroscopic manifestation.

It seems that the main obstacles in the way of superconductivity's applications in practice are not the theoretical problems of its in-depth study, but more a purely

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[1] I. Kikoine's study of the gyromagnetic effect in the superconductor in the early 1930s led him to determination of the gyromagnetic factor of the superconducting carriers.

technological problem. In short, working with liquid helium is too time-consuming and costly and also the technology of nitrogen level superconductors has not yet mastered.

The main problem still lies in correct understanding the physics of superconductivity. Of course, R. Kirchhoff was correct saying that there is nothing more practical than good theory. Therefore, despite the obvious and critical importance of the issues related to the application of superconductors and challenges faced by their technology, they will not be considered.

The most important task of the fundamental theory of superconductivity is to understand the physical mechanisms forming the superconducting state. That is, to find out how the basic parameters of superconductors depend on other physical properties. We also need to overcome a fact that the current theory of superconductivity could not explain ł why some superconductors have been observed at critical temperature in a critical field.

These were the facts and concepts that have defined our approach to this consideration.

In the first part of it, we focus on the steps made to understand the phenomenon of superconductivity and the problems science has encountered along the way.

The second part of our consideration focuses on the explanation of superconductivity, which has been described as the consequence of ordering of zero-point fluctuations of electrons and that are in satisfactory agreement with the measured data.

The phenomenon of superfluidity in He-4 and He-3 can be similarly explained, due to ordering of zero-point fluctuations.

Thus, it is important that both related phenomena are based on the same physical mechanism.

3.2   Discovery of Superconductivity

Figure 3.1   *Heike Kamerling-Onnes.*

At the beginning of the twentieth century, a new form of scientific research of appeared in the world. Heike Kamerling-Onnes was one of first scientists who used the industry for the service of physics. His research laboratory was based on the present plant of freeze consisting of refrigerators which he developed. This industrial approach gave him complete benefits of the world monopoly in studies at low temperatures for a long time (15 years!). Above all, he was able to carry out his solid-state studies at liquid helium (which boils at atmospheric pressure at 4.18 K). He was the first who creates liquid helium in 1908, then he began his systematic studies of the electrical resistance of metals. It was known from earlier experiments that the electrical resistance of metals decreases with decreasing temperature. Moreover, their residual resistance turned out to be smaller if the metal was cleaner. So the idea arose to measure this dependence in pure platinum and gold. But at that time, it was impossible to get these metals sufficiently clean. In those days, only mercury could be obtained at a very high degree of purification by method of repeated distillation. The researchers were lucky. The superconducting transition in mercury occurs at 4.15K, i.e. slightly below the boiling point of helium. This has created sufficient conditions for the discovery of superconductivity in the first experiment.

One hundred years ago, at the end of November 1911, Heike Kamerlingh Onnes submitted the article [16] where remarkable phenomenon of the complete disappearance of electrical resistance of mercury, which he called superconductivity, was described. Shortly thereafter, thanks to the evacuation of vapor, H. Kamerling-Onnes and his colleagues discovered superconductivity in tin and then in other metals, that were not necessarily very pure. It was therefore shown that the degree of cleanliness has little effect on the superconducting transition.

Their discovery concerned the influence of magnetic fields on superconductors. They therefore determined the existence of the two main criteria of superconductors: the critical temperature and the critical magnetic field.[2]

The physical research at low temperature was started by H. Kamerling-Onnes and has now been developed in many laboratories around the world.

But even a hundred years later, the general style of work in the Leiden cryogenic laboratory created by H. Kamerling-Onnes, including the reasonableness of its scientific policy and the power of technical equipment, still impress specialists.

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[2] Nobel Laureate V. L. Ginzburg gives in his memoirs the excellent description of events related to the discovery of superconductivity. He drew special attention to the ethical dimension associated with this discovery. Ginsburg wrote [17]: "The measurement of the mercury resistance was held Gilles Holst. He was first who observed superconductivity in an explicit form. He was the qualified physicist (later he was the first director of Philips Research Laboratories and professor of Leiden University). But his name in the Kamerling-Onnes's publication is not even mentioned. As indicated in [18], G.Holst itself, apparently, did not consider such an attitude Kamerling-Onnes unfair and unusual. The situation is not clear to me, for our time that is very unusual, perhaps 90 years ago in the scientific community mores were very different."

Chapter 4

Basic Milestones in the Study of Superconductivity

The first twenty two years after the discovery of superconductivity, only the Leiden laboratory of H. Kamerling-Onnes engaged in its research. Later helium liquefiers began to appear in other places, and other laboratories were began to study superconductivity. The significant milestone on this way was the discovery of absolute diamagnetism effect of superconductors. Until that time, superconductors were considered as ideal conductors. W. Meissner and R. Ochsenfeld [19] showed in 1933, that if a superconductor is cooled below the critical temperature in a constant and not very strong magnetic field, then this field is pushed out from the bulk of superconductor. The field is forced out by undamped currents that flow across the surface.[1]

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[1] Interestingly, Kamerling-Onnes was searching for this effect and carried out the similar experiment almost twenty years earlier. The liquefaction of helium was very difficult at that time so he was forced to save on it and used a thin-walled hollow ball of lead in his measurements. It is easy to "freeze" the magnetic field in thin-walled sphere and with that the Meissner effect would be masked.

4.1   The London Theory

Figure 4.1   _Brothers Heinz and Fritz London._

The great contribution to the development of the science of superconductors was made by brothers Fritz and Heinz London. They offered its first phenomenological theory. Before the discovery of the absolute diamagnetism of superconductors, it was thought that superconductors are absolute conductors, or in other words, just metals with zero resistance. At a first glance, there is no particulary difference in these definitions. If we consider a perfect conductor in a magnetic field, the current will be induced onto its surface and will extrude the field, i.e. diamagnetism will manifest itself. But if at first we magnetize the sample by placing it in the field, then it will be cooled, diamagnetism should not occur. However, in accordance with the Meissner-Ochsenfeld effect, the result should not depend on the sequence of the vents. Inside superconductors the resistance is always:

$$\rho = 0, \tag{4.1}$$

and the magnetic induction:

$$B = 0. \tag{4.2}$$

In fact, the London theory [20] is the attempt to impose these conditions on Maxwells equations.

The consideration of the London penetration depth is commonly accepted (see for example [22]) in several steps:

Step 1. Firstly, the action of an external electric field on free electrons is considered. In accordance with Newton's law, free electrons gain acceleration in an electric field $\mathbf{E}$:

$$\mathbf{a} = \frac{e\mathbf{E}}{m_e}. \tag{4.3}$$

The directional movement of the "superconducting" electron gas with the density n_s creates the current with the density:

$$\mathbf{j} = e n_s \mathbf{v}, \tag{4.4}$$

where $\mathbf{v}$ is the carriers velocity. After differentiating the time and substituting this in Eq.(4.3), we obtain the first London equation:

$$\frac{d}{dt}\mathbf{j} = e n_s \mathbf{a} = \frac{n_s e^2}{m_e}\mathbf{E}. \tag{4.5}$$

Step 2. After application of operations rot to both sides of this equation and by using Faradays law of electromagnetic induction $rot\,\mathbf{E} = -\frac{1}{c}\frac{d\mathbf{B}}{dt}$, we then acquire the relationship between the current density and magnetic field:

$$\frac{d}{dt}\left(rot\,\mathbf{j} + \frac{n_s e^2}{m_e c}\mathbf{B}\right) = 0. \tag{4.6}$$

Step 3. By selecting the stationary solution of Eq.(4.6)

$$rot\,\mathbf{j} + \frac{n_s e^2}{m_e c}\mathbf{B} = 0, \tag{4.7}$$

and after some simple transformations, one can conclude that there is a so-called London penetration depth of the magnetic field in a superconductor:

$$\Lambda_L = \sqrt{\frac{m_e c^2}{4\pi e^2 n_s}}. \tag{4.8}$$

The London penetration depth and the density of superconducting carriers. One of the measurable characteristics of superconductors is the London penetration depth, and for many of these superconductors it usually equals to a few hundred Angstroms [23]. In the Table 4.1 the measured values of λ_L are given in the second column.

Table 4.1 *The London penetration depth and the density of superconducting carriers.*

super-conductors	$\lambda_L, 10^{-6}$cm measured [23]	n_s according to Eq.(4.8)	n_e in accordance with Eq.(7.27)	n_s/n_e
Tl	9.2	$3.3 \cdot 10^{21}$	$1.4 \cdot 10^{23}$	0.023
In	6.4	$6.9 \cdot 10^{21}$	$3.0 \cdot 10^{23}$	0.024
Sn	5.1	$1.1 \cdot 10^{22}$	$3.0 \cdot 10^{23}$	0.037
Hg	4.2	$1.6 \cdot 10^{22}$	$4.5 \cdot 10^{22}$	0.035
Pb	3.9	$1.9 \cdot 10^{22}$	$1.0 \cdot 10^{24}$	0.019

If we are to use this experimental data to calculate the density of superconducting carriers n_s in accordance with the Eq.(4.8), the results would about two orders of magnitude larger (see the middle column of Table 4.1.

Only a small fraction of these free electrons can combine into the pairs. This is only applicable to the electrons that energies lie within the thin strip of the energy spectrum near $\mathcal{E}_F$. We can therefore expect that the concentration of superconducting carriers among all free electrons of the metal should be at the level $\frac{n_s}{n_e} \approx 10^{-5}$ (see Eq.(7.23)). These concentrations, if calculated from Eq.(4.8), are seen to be about two orders of magnitude higher (see last column of the Table 4.1). Apparently, the reason for this discrepancy is because of the use of a nonequivalent transformation. At the first stage in Eq.(4.3), the straight-line acceleration in a static electric field is considered. If this moves, there will be no current circulation. Therefore, the application of the operation rot in Eq.(4.6) in this case is not correct. It does not lead to Eq.(4.7):

$$\frac{rot \, \mathbf{j}}{\frac{n_s e^2}{m_e c} \mathbf{B}} = -1, \tag{4.9}$$

but instead, leads to a pair of equations:

$$rot\,\mathbf{j} = 0$$

$$\frac{n_s e^2}{m_e c}\mathbf{B} = 0 \tag{4.10}$$

and to the uncertainty:

$$\frac{rot\,\mathbf{j}}{\frac{n_s e^2}{m_e c}\mathbf{B}} = \frac{0}{0}. \tag{4.11}$$

The correction of the ratio of the Londons depth with the density of superconducting carriers will be given in section (9).

4.2   The Ginsburg-Landau Theory

The London phenomenological theory of superconductivity does not account for the quantum effects.

The theory proposed by V. L. Ginzburg and L. D. Landau [24] in the early 1950s, uses the mathematical formalism of quantum mechanics. Nevertheless, it is a phenomenological theory, since it does not investigate the nature of superconductivity, although it qualitatively and quantitatively describes many aspects of characteristic effects.

To describe the motion of particles in quantum mechanics one uses the wave function $\Psi(r, t)$, which characterizes the position of a particle in space and time. In the GL-theory, such a function is introduced to describe the entire ensemble of particles and is named the parameter of order. Its square determines the concentration of the superconducting particles.

At its core, the GL-theory uses the apparatus, which was developed by Landau, to describe order-disorder transitions (by Landaus classification, it is transitions of the second kind). According to Landau, the transition to a more orderly system should be accompanied by a decrease in the amount of free energy:

$$\Delta W = -a \cdot n_s + \frac{b}{2}n_s^2, \tag{4.12}$$

where a and b are model parameters. Using the principle of minimum free energy of the system in a steady state, we can find the relation between these parameters:

$$\frac{d(\Delta W)}{dn_s} = -a + b \cdot n_s = 0. \tag{4.13}$$

Whence

$$b = \frac{a}{n_s} \tag{4.14}$$

and the energy gain in the transition to an ordered state:

$$\Delta W = -\frac{a}{2} n_s. \tag{4.15}$$

The reverse transition from the superconducting state to a normal state occurs at the critical magnetic field strength, H_c. This is required to create the density of the magnetic energy $\frac{H_c^2}{8\pi}$. According to the above description, this equation is therefore obtained:

$$\frac{H_c^2}{8\pi} = \frac{a}{2} n_s. \tag{4.16}$$

In order to express the parameter a of GL-theory in terms of physical characteristics of a sample, the density of "superconducting" carriers generally charge from the London's equation (4.8).[2]

The important step in the Ginzburg-Landau theory is the changeover of the concentration of superconducting carriers, n_s, to the order parameter Ψ

$$|\Psi(x)|^2 = n_s. \tag{4.17}$$

At this the standard Schrodinger equation (in case of one dimension) takes the form:

$$-\frac{\hbar}{2m} \left[\nabla \Psi(x) \right]^2 - a\Psi^2(x) + \frac{b}{2}\Psi^4(x) = \mathcal{E}. \tag{4.18}$$

Again using the condition of minimum energy

$$\frac{d\mathcal{E}}{d\Psi} = 0 \tag{4.19}$$

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[2] It should be noted that due to the fact that the London equation does not correctly describes the ratio of the penetration depth with a density of carriers, one should used the revised equation (9.13) in order to find the a.

after the simple transformations one can obtain the equation that is satisfied by the order parameter of the equilibrium system:

$$a\Psi + b\Psi|\Psi|^2 + \frac{1}{4m_e}\left(i\hbar\nabla + \frac{2e}{c}\mathbf{A}\right)^2\Psi = 0. \tag{4.20}$$

This equation is called the first Ginzburg-Landau equation. It is nonlinear. Although there is no analytical solution for it, by using the series expansion of parameters, we can find solutions to many of the problems which are associated with changing the order parameter. Such there are consideration of the physics of thin superconducting films, boundaries of superconductor-metal, phenomena near the critical temperature and so on. The variation of the Schrodinger equation (4.18) with respect the vector potential $\mathbf{A}$ gives the second equation of the GL-theory:

$$\mathbf{j}_s = \frac{i\hbar e}{2m_e}\left(\Psi^*\nabla\Psi - \Psi\nabla\Psi^*\right) - \frac{2e^2}{m_e c}|\Psi|^2\mathbf{A}. \tag{4.21}$$

This determines the density of superconducting current. This equation allows us to obtain a clear picture of the important effect of superconductivity: the magnetic flux quantization.

4.3   Experimental Data That Are Important for Creation of the Theory of Superconductivity

4.3.1   Features of the Phase Transition

Phase transitions can occur with a jump of the first derivatives of thermodynamic potential and with a jump of second derivatives at the continuous change of the first derivatives. In the terminology of Landau, there are two types of phase transitions: the 1st and the 2nd types. Phenomena with rearrangement of the crystal structure of matter are considered to be a phase transition of the 1st type, while the order-disorder transitions relate to the 2nd type. Measurements show that at the superconducting transition there are no changes in the crystal structure or the latent heat release and similar phenomena characteristic of first-order transitions. On the contrary, the specific

heat at the point of this transition is discontinuous (see below). These findings clearly indicate that the superconducting transition is associated with a change order. The complete absence of changes of the crystal lattice structure, proven by X-ray measurements, suggests that this transition occurs as an ordering in the electron subsystem.

4.3.2 The Energy Gap and Specific Heat of a Superconductor

The energy gap of a superconductor. Along with the X-ray studies that show no structural changes at the superconducting transition, no changes can be seen in the optical range. When viewing with the naked eye here, nothing happens. However, the reflection of radio waves undergoes a significant change in the transition. Detailed measurements show that there is a sharp boundary in the wavelength range 1 1011 5 1011 Hz, which is different for different superconductors. This phenomenon clearly indicates on the existence of a threshold energy, which is needed for the transition of a superconducting carrier to normal state, i.e., there is an energy gap between these two states.

The specific heat of a superconductor. The laws of thermodynamics provide possibility for an idea of the nature of the phenomena by means of general reasoning. We show that the simple application of thermodynamic relations leads to the conclusion that the transition of a normal metal-superconductor transition is the transition of second order, i.e., it is due to the ordering of the electronic system.

In order to convert the superconductor into a normal state, we can do this via a critical magnetic field, H_c. This transition means that the difference between the free energy of a bulk sample (per unit of volume) in normal and superconducting states complements the energy density of the critical magnetic field:

$$F_n - Fs = \frac{H_c^2}{8\pi}. \tag{4.22}$$

By definition, the free energy is the difference of the internal energy, U, and thermal energy TS (where S is the entropy of a state):

$$F = U - TS. \tag{4.23}$$

Therefore, the increment of free energy is

$$\delta F = \delta U - T\delta S - S\delta T. \tag{4.24}$$

According to the first law of thermodynamics, the increment of the density of thermal energy δQ is the sum of the work made by a sample on external bodies δR, and the increment of its internal energy δU:

$$\delta Q = \delta R + \delta F \tag{4.25}$$

as a reversible process heat increment of $\delta Q = T\delta S$, then

$$\delta F = -\delta R - S\delta T \tag{4.26}$$

thus the entropy

$$S = -\left(\frac{\partial F}{\partial T}\right)_R. \tag{4.27}$$

In accordance with this equation, the difference of entropy in normal and superconducting states (4.22) can be written as:

$$S_s - S_n = \frac{H_c}{4\pi}\left(\frac{\partial H_c}{\partial T}\right)_R. \tag{4.28}$$

Since critical field at any temperature decreases with rising temperature:

$$\left(\frac{\partial H_c}{\partial T}\right) < 0, \tag{4.29}$$

then we can conclude (from equation (4.28)), that the superconducting state is more ordered and therefore its entropy is lower. Besides this, since at $T = 0$, the derivative of the critical field is also zero, then the entropy of the normal and superconducting state, at this point, are equal. Any abrupt changes of the first derivatives of the thermodynamic potential must also be absent, i.e., this transition is a transition of the order-disorder in electron system.

Since, by definition, the specific heat $C = T\left(\frac{\partial S}{\partial T}\right)$, then the difference of specific heats of superconducting and normal states:

$$C_s - C_n = \frac{T}{4\pi}\left[\left(\frac{\partial H_c}{\partial T}\right)^2 + H_c\frac{\partial^2 H_c}{\partial T^2}\right]. \tag{4.30}$$

Since at the critical point $H_c = 0$, then from (4.30) this follows directly the Rutgers formula that determines the value of a specific heat jump at the transition point:

$$C_s - C_n = \frac{T}{4\pi} \left(\frac{\partial H_c}{\partial T} \right)^2_{T_c}.$$ (4.31)
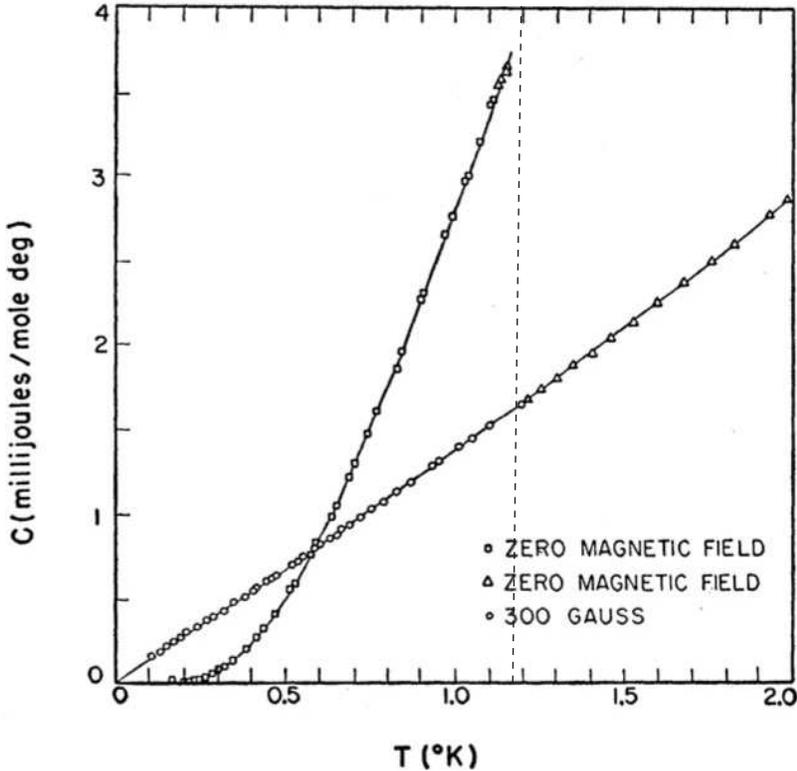
Figure 4.2 *Low-temperature heat capacity of normal and superconducting aluminum[25].*

The theory of the specific heat of superconductors is well-confirmed experimentally. For example, the low-temperature specific heat of aluminum in both the superconducting and normal states supports this in Figure 4.2. Only the electrons determine the heat

capacity of the normal aluminum at this temperature, and in accordance with the theory of Sommerfeld, it is linear in temperature. The specific heat of a superconductor at a low temperature is exponentially dependent on it. This indicates the existence of a two-tier system in the energy distribution of the superconducting particles. The measurements of the specific heat jump at T_c is well described by the Rutgers equation (4.31).

4.3.3   Magnetic Flux Quantization in Superconductors

The conclusion that magnetic flux in hollow superconducting cylinders should be quantized was firstly expressed F.London. However, the main interest in this problem is not in the phenomenon of quantization, but in the details: what should be the value of the flux quantum. F. London had not taken into account the effect of coupling of superconducting carriers when he computed the quantum of magnetic flux and, therefore, predicted for it twice the amount. The order parameter can be written as:

$$\Psi(r) = \sqrt{n_s}\, e^{i\theta(r)}. \tag{4.32}$$

where n_s is density of superconducting carriers, θ is the order parameter phase.

As in the absence of a magnetic field, the density of particle flux is described by the equation:

$$n_s \mathbf{v} = \frac{i\hbar}{2m_e} \left(\Psi \nabla \Psi^* - \Psi^* \nabla \Psi \right). \tag{4.33}$$

Using (4.32), we get $\hbar \nabla \theta = 2m_e \mathbf{v_s}$ and can transform the Ginzburg-Landau equation (4.21) to the form:

$$\hbar \nabla \theta = 2m_e \mathbf{v}_s + \frac{2e}{c} \mathbf{A}. \tag{4.34}$$

If we consider the freezing of magnetic flux in a thick superconducting ring with a wall which thickness is much larger than the London penetration depth λ_L, in the depths of the body of the ring current density of $\mathbf{j}$ is zero. This means that the equation (4.34) reduces to the equation:

$$\hbar \nabla \theta = \frac{2e}{c} \mathbf{A}. \tag{4.35}$$

One can take the integrals on a path that passes in the interior of the ring, not going

close to its surface at any point, on the variables included in this equation:

$$\hbar \oint \nabla \theta ds = \frac{2e}{c} \oint \mathbf{A} ds, \tag{4.36}$$

and obtain

$$\oint \nabla \theta ds = \frac{2e}{\hbar c} \Phi, \tag{4.37}$$

since by definition, the magnetic flux through any loop:

$$\Phi = \oint \mathbf{A} ds. \tag{4.38}$$

The contour integral $\oint \nabla \theta ds$ must be a multiple of 2π, to ensure the uniqueness of the order parameter in a circuit along the path. Thus, the magnetic flux trapped by superconducting ring should be a multiple to the quantum of magnetic flux:

$$\Phi_0 = \frac{2\pi \hbar c}{2e}, \tag{4.39}$$

that is confirmed by corresponding measurements.

This is a very important result for understanding the physics of superconductivity. Thus, the theoretical predictions are confirmed by measurements saying that the superconductivity is due to the fact that its carriers have charge 2e, i.e., they represent two paired electrons. It should be noted that the pairing of electrons is a necessary condition for the existence of superconductivity, but this phenomenon was observed experimentally in the normal state of electron gas metal too. The value of the quantum Eq.(36) correctly describes the periodicity of the magnetic field influence on electron gas in the normal state of some metals (for example, Mg and Al at temperatures much higher than their critical temperatures) [41], [42]).

4.3.4 The Isotope Effect

The most important yet negative role, which plays a major part in the development of the science of superconductivity, is the isotope effect, which was discovered in 1950. The negative role, of the isotope effect is played not just by the effect itself but its wrong interpretation. It was established through an experiment that the critical temperatures of

superconductors depend on the isotope mass Mi, from which they are made:

$$T_c \sim \frac{1}{M_i^a}.$$ (4.40)

This dependence was called the isotope effect. It was found that for type-I superconductors - Zn, Sn, In, Hg, Pb - the value of the isotope effect can be described by Eq.(4.40) at the constant $a = \frac{1}{2}$.
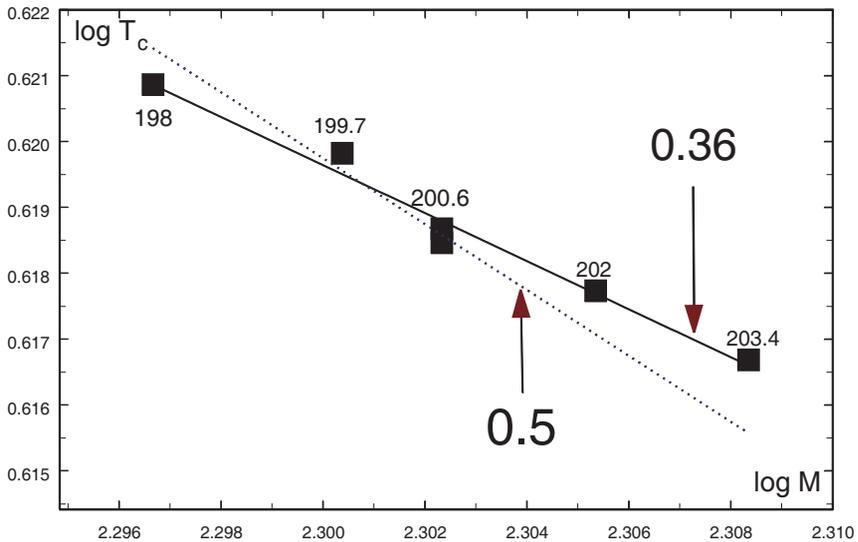
Figure 4.3   *The isotope effect in mercury. The solid line is obtained by the sparse-squares technique. In accordance with the phonon mechanism, the coefficient а must be about 1/2 (the dotted line). As it can be seen, this coefficient is in reality approximately equal to 1/3.*

This effect has made researchers think that the phenomenon of superconductivity is actually associated with the vibrations of ions in the lattice. This is because of the similar dependence (Eq.(4.40)) on the ion mass in order for the maximum energy of phonons to exist whilst propagating in the lattice.

Subsequently this simple picture was broken: the isotope effect was measured for other metals, and it had different values. This difference of the isotope effect in different superconductors could not be explained by phonon mechanism.

It should be noted that the interpretation of the isotope effect in the simple metals did exist though it seemed to fit the results of measurements under the effect of phonons, where $a = 1/2$. Since the analysis of experimental data [35], [36] (see Figure 4.3) suggests that this parameter for mercury is really closer to 1/3.

4.4　BCS

The first attempt to detect the isotope effect in lead was made by the Leiden group in the early 1920s, but due to a smallness of the effect it was not found. It was then registered in 1950 by researchers of the two different laboratories. It created the impression that phonons are responsible for the occurrence of superconductivity since the critical parameters of the superconductor depends on the ion mass. In the same year H.Fröhlich was the first to point out that at low temperatures, the interaction with phonons can lead to nascency of forces of attraction between the electrons, in spite of the Coulomb repulsion. A few years later, L. Cooper predicted the specific mechanism in which an arbitrarily weak attraction between electrons with the Fermi energy would lead to the emergence of a bound state. On this basis, in 1956, Bardeen, Cooper and Shrieffer built a microscopic theory, based on the electron-phonon interaction as the cause of the attractive forces between electrons.

Figure 4.4 *John Bardeen - twice winner of the Nobel Prize. He received the Prize in 1956 for the invention of the transistor, and in 1972, along with L. Cooper and J. Shrieffer, for the creation of the BCS-theory. I would like to put here somewhere my photo at the conversation with John Bardeen. It would be a remarkable illustration of the continuity of generations within the science of superconductivity! The great thing that this photo must exist. In the late 1980s, I was at the conference on superconductivity at Stanford University. I met J. Bardeen, who also attended the conference. It was there that I spoke to him. While we were talking, I saw an American physicist taking a photo of us. I knew this physicist at the time, because he had attend my laboratory in Dubna to acquaint himself with the work of high-Tc SQUID. Our laboratory gained international recognition, because we were the first scientific team in the world who could overcome the natural barrier of SQUID sensitivity [31]. This work was aimed at measuring the magnetic cardiogram of humans with the help of high-Tc SQUID. After twenty-odd years now though, I cannot remember neither the name of this American scientist, nor even where he had come from. I hope the photographs taken during his visit to Dubna still exist. So there is still hope that with the help of American friends I can find him, and with him, those photos made more than twenty years ago.*

It is believed that the BCS-theory [13] has the following main results:

1. The attraction in the electron system arises due to the electron-phonon interaction. As result of this attraction, the ground state of the electron system is separated from the excited electrons by an energetic gap. The existence of energetic gap explains the behavior of the specific heat of superconductors, optical experiments and so on.

2. The depth of penetration (as well as the coherence length) appears to be a natural consequence of the ground state of the BCS-theory. The London equations and the Meissner diamagnetism are obtained naturally.

3. The criterion for the existence of superconductivity and the critical temperature of the transition involves itself the density of electronic states at the Fermi level $\mathcal{D}(\mathcal{E}_F)$ and the potential of the electron-lattice interaction U, which can be estimated from the electrical resistance. In the case of $U\mathcal{D}(\mathcal{E}_F) \ll 1$ the BCS-theory expresses the critical temperature of the superconductor in terms of its Debye temperature Θ_D:

$$T_c = 1.14 \cdot \Theta_D \cdot exp\left[-\frac{1}{U\mathcal{D}(\mathcal{E}_F)}\right]. \tag{4.41}$$

4. The participation of the lattice in the electron-electron association determines the effect of isotopic substitution on the critical temperature of the superconductor. At the same time due to the fact that the mass of the isotopes depends on the Debye temperature $\theta_D \propto M^{-1/2}$, Eq.(4.41) correctly describes this relationship for a number of superconductors.

5. The temperature dependence of the energy gap of the superconductor is described in the BCS-theory implicitly by an integral over the phonon spectrum from 0 to the Debye energy:

$$1 = \frac{U\mathcal{D}(\mathcal{E}_F)}{2}\int_0^{\hbar\omega_D} d\xi \frac{th\sqrt{\xi^2 + \Delta^2}/2kT}{\sqrt{\xi^2 + \Delta^2}}. \tag{4.42}$$

The result of calculation of this dependence is in good agreement with measured data (Figure 4.5).
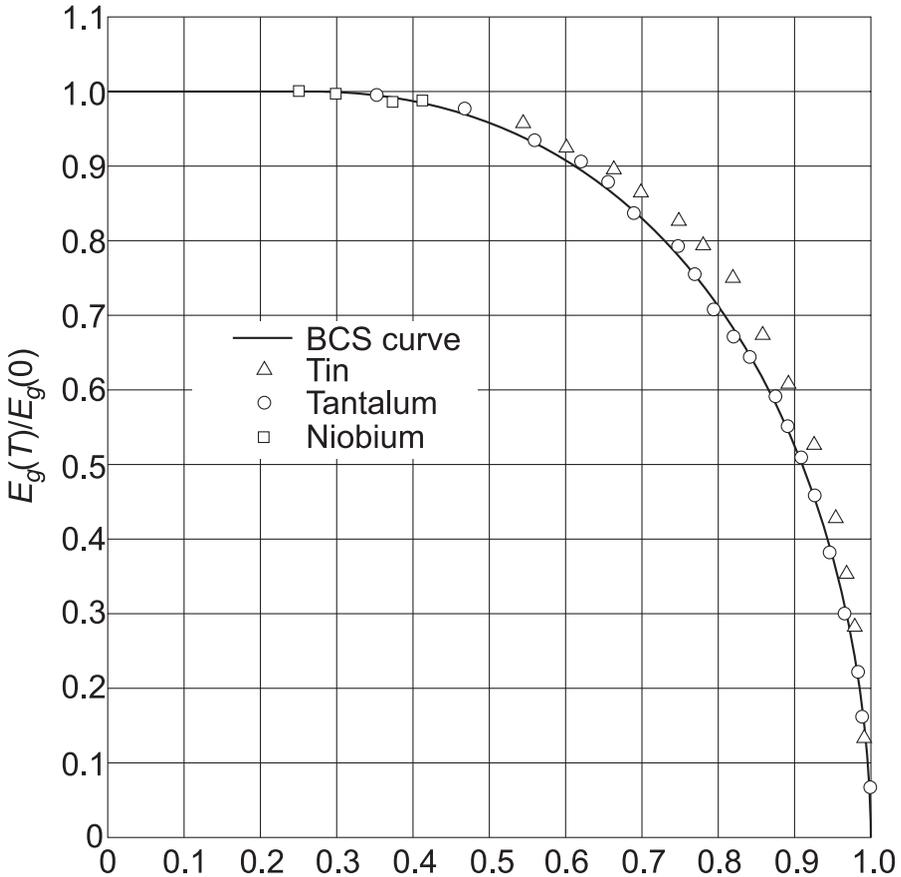
Figure 4.5   *The temperature dependence of the gap in the energy spectrum of superconductors, calculated by the Eq.(4.42).*

6. The BCS-theory is consistent with the data of measurements of the magnetic flux quantum, as its ground state is made by pairs of single-electron states.

But all this is not in a good agreement with this theory.

First, it does not give the main answer to our questions: with using of it one cannot obtain apriori information about what the critical parameters of a particular superconductor should be. Therefore, BCS cannot help in the search for strategic development of superconductors or in the tactics of their research. Eq.(4.41) contains two parameters that are difficult to assess: the value of the electron-phonon interaction and the density of electronic levels near the Fermi level. Therefore, BCS cannot be used

solely for this purpose, and can only give a qualitative assessment.

In addition, many results of the BCS theory can be obtained by using other, simpler but 'fully correct' methods.

The coupling of electrons in pairs can be the result not only of electron-phonon mechanism. Any attraction between the electrons can lead to their coupling.

For the existence of superconductivity, the bond energy should combine into single ensembles of separate pairs of electrons, which are located at distances of approximately hundreds of interatomic distances. In BCS-theory, there are no forces of attraction between the pairs and, especially, between pairs on these distances.

The quantization of flux is well described within the Ginzburg-Landau theory (see Sec.(4.3.3)), if the order parameter describes the density of paired electrons.

By using a two-tier system with the approximate parameters, it is easy to describe the temperature dependence of the specific heat of superconductors.

So, the calculation of the temperature dependence of the superconducting energy gap formula (Eq.(4.42)) can be considered as the success of the BCS theory .

However, it is easier and more convenient to describe this phenomenon as a characterization of the order-disorder transition in a two-tier system of zero-point fluctuations of condensate. In this approach, which is discussed below in Sec.(7.2), the temperature dependence of the energy gap receives the same interpretation as other phenomena of the same class ł such as the λ-transition in liquid helium, the temperature dependence of spontaneous magnetization of ferromagnets and so on.

Therefore, as in the 1950s, the existence of isotope effect is seen to be crucial. However, to date, there is experimental evidence that shows the isotope substitution leads to a change of the parameters of the metals crystal lattice due to the influence of isotope mass on the zero-point oscillations of the ions (see [50]). For this reason, the isotope substitution in the lattice of the metal should lead to a change in the Fermi energy and its impact on all of its electronic properties. In connection with this, the changing of the critical temperature of the superconductor at the isotope substitution can be a direct consequence of changing the Fermi energy without any participation of phonons.

The second part of this book will be devoted to the role of the ordering of zero-point oscillations of electrons in the mechanism of the superconducting state formation.
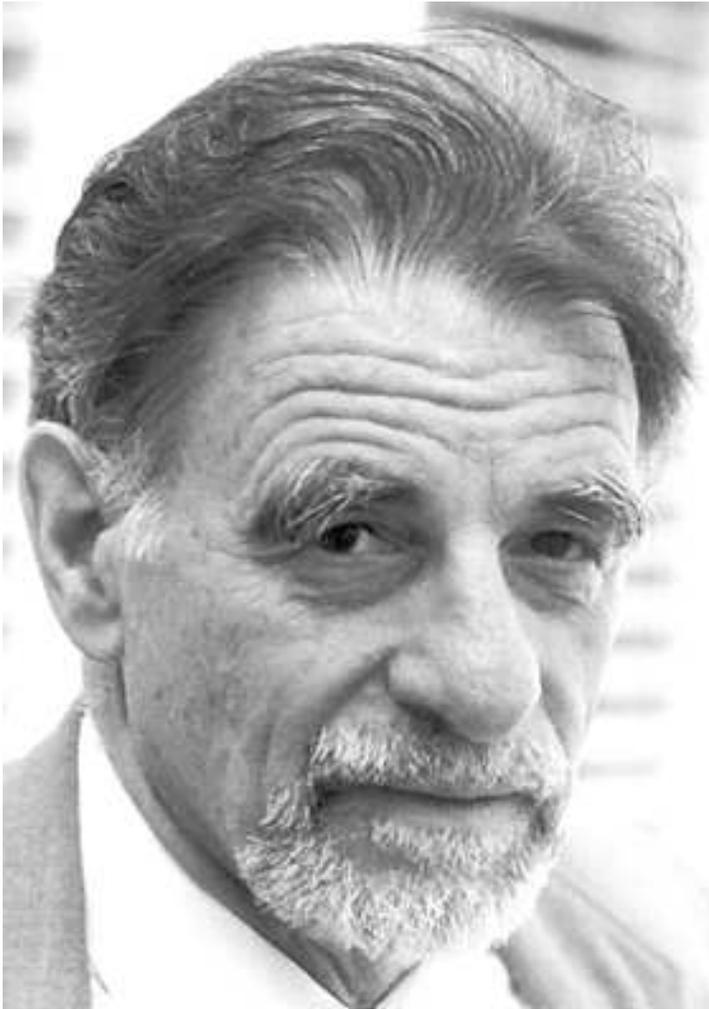
Figure 4.6 *Karl Alexander Muller - founder of HTSC.*

4.5   The New Era - HTSC

During the century following the discovery of superconductivity, 40 pure metals were observed. It turned out that among them, Magnesium has the lowest critical temperature ł of about 0.001K, and Technetium has the highest at 11.3K.

Also, it was found that hundreds of compounds and alloys at low temperatures have the property of superconductivity. Among them, the intermetallic compound Nb_3Ge has the highest critical temperature ł 23.2K.

In order to obtain the superconducting state in these compounds it is necessary to use the expensive technology of liquid helium.[3]

Theoretically, it seems that liquid hydrogen could also be used in some cases. But this point is still more theoretical consideration than practical one: hydrogen is a very explosive substance.   For decades scientists have nurtured a dream to create a superconductor which would retain its properties at temperatures above the boiling point of liquid nitrogen. Liquid nitrogen is cheap, accessible, safe, and a subject to a certain culture of working with him (or at least it is not explosive).   The creation of such superconductor promised breakthrough in many areas of technology.

In 1986, these materials were found.   At first, Swiss researchers A. Muller and G. Bednortz found the superconductivity in the copper-lanthanum ceramics, which temperature of superconducting transition was only 40K, and soon it became clear that it was the new class of superconductors (they was called high-temperature superconductors, or HTSC), and a very large number of laboratories around the world were included in studies of these materials.

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3   For comparison, we can say that liter of liquid helium costs about a price of bottle of a good brandy, and the heat of vaporization of helium is so small that expensive cryostats are needed for its storage. That makes its using very expensive.

Table 4.2   *Critical parameters of superconductors.*

superconductor	T_c, K	H_c, Oe
Hg	4.15	41
Pb	7.2	80
Nb	9.25	206
$NbTi$	9.5-10.5	120.000
Nb_3Sn	18.1-18.5	220.000
Nb_3Al	18.9	300.000
Nb_3Ge	23.2	370.000
MgB_2	40	150.000
$YBa_2Cu_3O_7$	92.4	600.000
$Bi_2Sr_2Ca_2Cu_3O_{10}$	111	5.000.000
$HgBa_2Ca_2Cu_3O_8$	133	>10.000.000

One year later, the superconductivity was discovered in ceramic $YBaCuO$ with transition temperature higher than 90K. As the liquid nitrogen boils at 78K, the nitrogen level was overcome.

Soon after it, mercury based ceramics with transition temperatures of the order of 140K were found. The history of increasing the critical temperature of the superconductor is interesting to trace: see the Figure 4.7.
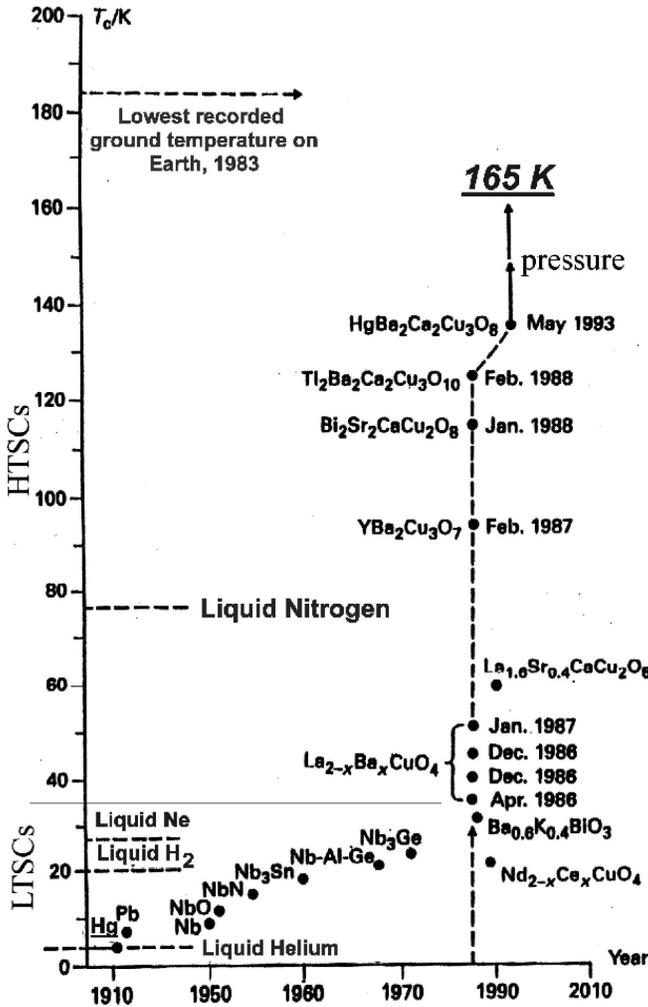
Figure 4.7 *Growth of the critical temperature of known superconductors in time.*
The horizontal lines indicate the temperature of boiling of cryogenic liquids
(at normal pressure).

It is clear from this graph that if the creation of new superconductors has been continuing at the same rate as before the discovery of HTSC, the nitrogen levels would have been overcome through 150 years. But science is developing by its own laws, and the discovery of HTSC allowed to raise sharply the critical temperature.

However, the creation of high-Tc superconductors has not led to a revolutionary breakthrough in technology and industry. Ceramics were too low-technological to manufacture a thin superconducting wires. Without wires, the using of high-Tc superconductors had to be limited by low-current instrument technology. It also did not cause the big breakthroughs in this manner, either (see, e.g. [31]).

After discoveries of high-Tc superconductors, no new fundamental breakthrough of this value was made. Perhaps the reason for this is that the BCS-theory, adopted by most researchers to date, can not predict the parameters of superconductors apriori and serves as just a support in strategy and tactics of their research.

Chapter 5

Superfluidity

The first study of the properties of helium-II began in the Leiden laboratory as early as in 1911, the same year when superconductivity was discovered. A little later the singularity in the specific heat, called the λ-transition, was discovered. However, the discovery of superfluidity of liquid helium was still far away, as Pyotr L. Kapitsa discovered it by 1938.

This discovery became a landmark for the world science, so many events associated with it are widely known, but one story of Kapitsa's background was never published.

This Soviet scientist Isaak K. Kikoine relayed this story to me in the mid 1960s, at this time, when I was his graduate student. Kikoine was one of the leaders of the Soviet atomic project and was also engaged with important state affairs for most of the days. In the evenings, however, he often visited the labs of my colleagues or my laboratory to discuss scientific news. During these talks, he often intertwined scientific debate and interesting memories from history of physics.

Here is his story about Kapitsa and superfluidity as it was remembered and relayed to me.

Figure 5.1 *Pyotr L. Kapitsa.*

Figure 5.2 *Isaak K. Kikoine.*

It happened in 1933 when Isaac Kikoine was 25 years old. He had just completed his experiment of measuring of the gyromagnetic effect in superconductors. Pyotr Kaptsa was aware of this experiment, he even sent a small ball of super-pure lead for this experiment all the way from the Mond Laboratory in Cambridge, which he led then.

Almost every summer, he went with his family on his own car to Crimea. This was an absolute luxury for Soviet people at that time! On his way he visited physical laboratories in Moscow and Leningrad (now again St. Petersburg), lecturing and networking with colleagues, friends and admirers. During one of these visits, in 1933, Kikoine had a chance to talk to Kapitsa about results of his measurements. Kapitsa liked these results, and he invited Kikoine to work in Cambridge. They had arranged all formalities, including that Kapitsa will send him an invitation to work in England must be organized for the year. They planned to go to Cambridge together after the next summer vacation. But it has not happen. Summer 1934 Kapitsa as usual came to Russia. But when he wanted to go back to England, his return visa was canceled. No efforts helped him as it had been decided by the authorities at the top. Kapitsa's father-in-law was A. N. Krylov was a famous ship-builder. Krylov together with his friend Ivan Pavlov (Nobel Laureate in physiology from pre-revolutionary times) asked for an audience with Stalin himself. Stalin did receive them and asked them:

- What is the problem?

- Yesłoh, we are asking you to allow Kapitsa to go abroad.

- He will not be released. Because the Russian nightingale must sing in Russia! Pavlov (physiologist) said in reply:

- With all respect, a nightingale does not sing in a cage!

- Anyway he would sing for us!

So terribly unkind (it is a some understatement yet) Kapitsa became the deputy director of the Leningrad Physical-Technical Institute, which was directed by A. F. Ioffe. Young professors of this institution - Kikoine, Kurchatov, Alikhanov, Artsimovich then went to see the Deputy Director. Alikhanov asked at the door of the office from outgoing Artsimovich:

- Well, how is he? A beast or a man?

- A centaur! - was answer.

This nickname stuck firmly to P. Kapitsa. The scientists of the older generation called him by the same nickname, even decades later.

Stalin, of course, was a criminal. Thanks to his efforts, almost each family in the vast country lost some of its members in the terror of unjustified executions or imprisonments. However Stalin's role in the history of superfluidity can be considered positive. All was going well for Kapitsa in Russia, if to consider researches of superfluidity.
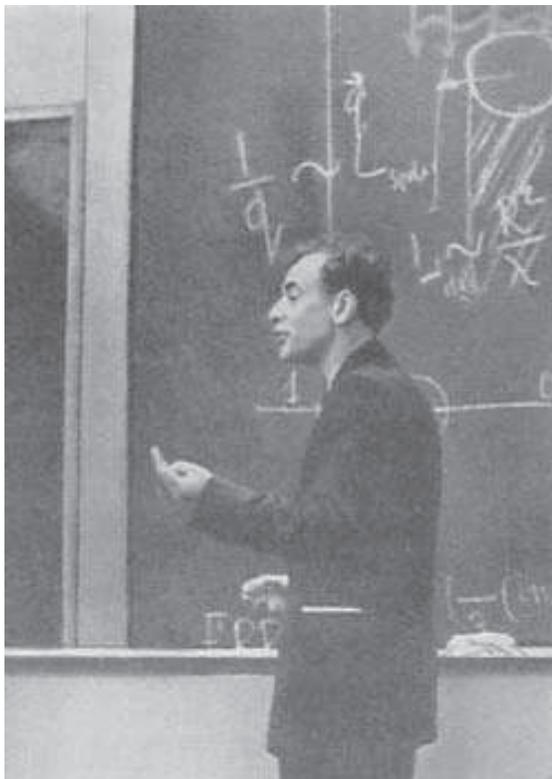
Figure 5.3 *L. D. Landau.*

Already after a couple of years, L. Landau[1] was able to give a theoretical explanation of this phenomenon. He viewed helium-II as a substance where the laws of quantum physics

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[1] Before P. Kapitsa spent a lot of the courage to pull out L. Landau from the stalinist prison.

worked on a macroscopic scale.

This phenomenon is akin to superconductivity: the superconductivity can be regarded as the superfluidity of an electron liquid. As a result, the relationship between phenomena have much in common, since both phenomena are described by the same quantum mechanics laws in macroscopic manifestations. This alliance, which exists in the nature of the phenomena, as a consequence, manifests itself in a set of physical phenomena: the same laws of quantum effects and the same physical description.

For example, even the subtle quantum effect of superconductors, such as the tunneling Josephson effect, manifests itself in the case of superfluidity as well.

However, there are some differences. At zero temperature, only a small number of all conduction electrons form the superfluid component in superconductors.

The concentration of this component is of the order $\frac{kT_c}{\mathcal{E}_F} \approx 10^{-5}$, while in liquid helium at $T = 0$, all liquid becomes superfluid, i.e., the concentration of the superfluid component is equal to 1.

It is also significant to note that the electrons which have an interaction with their environment have an electric charge and magnetic moment. This is the case yet it seems at first glance that there are no mechanisms of interaction for the formation of the superfluid condensate in liquid helium. Since the helium atom is electrically neutral, it has neither spin, nor magnetic moment.

By studying the properties of helium-II, it seems that all main aspects of the superfluidity has been considered. These include: the calculations of density of the superfluid component and its temperature dependence, the critical velocity of superfluid environment and its sound, the behavior of superfluid liquid near a solid wall and near the critical temperature and so on. These issues, as well as some others, are considered significantly in many high-quality original papers and reviews [27] - [30]. There is no need to rewrite their content here.

However, the electromagnetic mechanism of transition to the superfluid state in helium-4 still remains unclear, as it takes place at a temperature of about one Kelvin and also in the case of helium-3, at the temperature of about a thousand times smaller. It is obvious that this mechanism should be electromagnetic.

This is evidenced by the scale of the energy at which it occurs. The possible mechanism for the formation of superfluidity will be briefly discussed in the Chapter (11).

Superconductivity, Superfluidity and Zero-Point Oscillations

There is a parable that the population of one small south russian town in the old days was divided between parishioners of the Christian church and the Jewish synagogue.

A Christian priest was already heavily wiser by a life experience, and the rabbi was still quite young and energetic.

One day the rabbi came to the priest for advice.

- My colleague, - he said, - I lost my bike. I feel that it was stolen by someone from there, but who did it I can not identify. Tell me what to do.

- Yes, I know one scientific method for this case, - replied the priest.

You should do this: invite all your parishioners the synagogue and read them the ≪Ten Commandments of Moses≫. When you read ≪Thou shalt not steal≫, lift your head and look carefully into the eyes of your listeners. The listener who turns his eyes aside will be the guilty party.

A few days later, the rabbi comes to visit the priest with a bottle of Easter-vodka and on his bike. The priest asked the rabbi to tell him what happened in details. The rabbi told the priest that his theory had worked in practice and the bike was found. The rabbi relayed the story:

- I collected my parishioners and began to preach. While I approached reading the ≪Do not commit adultery≫ commandment I remembered then where I forgotten my bike!

So it is true: there is indeed nothing more practical than a good theory!

Chapter 6

Superconductivity as a Consequence of Ordering of Zero-Point Oscillations in Electron Gas

6.1 Superconductivity as a Consequence of Ordering of Zero-Point Oscillations

Superfluidity and superconductivity, which can be regarded as the superfluidity of the electron gas, are related phenomena. The main feature of these phenomena can be seen in a fact that a special condensate in superconductors as well as in superfluid helium is formed from particles interconnected by attraction. This mutual attraction does not allow a scattering of individual particles on defects and walls, if the energy of this scattering is less than the energy of attraction. Due to the lack of scattering, the condensate acquires ability to move without friction.

Superconductivity was discovered over a century ago, and the superfluidity about thirty years later.

However, despite the attention of many scientists to the study of these phenomena, they have been the great mysteries in condensed matter physics for a long time. This mystery attracted the best minds of the twentieth century.

The mystery of the superconductivity phenomenon has begun to drop in the middle of the last century when the effect of magnetic flux quantization in superconducting cylinders was discovered and investigated. This phenomenon was predicted even before the WWII by brothers F. London and H. London, but its quantitative study were performed only two decades later.

By these measurements it became clear that at the formation of the superconducting state, two free electrons are combined into a single boson with zero spin and zero momentum.

Around the same time, it was observed that the substitution of one isotope of the superconducting element to another leads to a changing of the critical temperature of superconductors: the phenomenon called an isotope-effect [35], [36]. This effect was interpreted as the direct proof of the key role of phonons in the formation of the superconducting state.

Following these understandings, L. Cooper proposed the phonon mechanism of electron pairing on which base the microscopic theory of superconductivity (so called BCS-theory) was built by N. Bogolyubov and J. Bardeen, L. Cooper and J. Shrieffer (probably it should be named better the Bogolyubov-BCS-theory).

However the B-BCS theory based on the phonon mechanism brokes a hypothetic link between superconductivity and superfluidity as in liquid helium there are no phonons for combining atoms.

Something similar happened with the description of superfluidity.

Soon after discovery of superfluidity, L. D. Landau in his first papers on the subject immediately demonstrated that this superfluidity should be considered as a result of condensate formation consisting of macroscopic number of atoms in the same quantum

state and obeying quantum laws. It gave the possibility to describe the main features of this phenomenon: the temperature dependence of the superfluid phase density, the existence of the second sound, etc. But it does not gave an answer to the question which physical mechanism leads to the unification of the atoms in the superfluid condensate and what is the critical temperature of the condensate, i.e. why the ratio of the temperature of transition to the superfluid state to the boiling point of helium-4 is almost exactly equals to $1/2$, while for helium-3, it is about a thousand times smaller.

On the whole, the description of both super-phenomena, superconductivity and superfluidity, to the beginning of the twenty first century induced some feeling of dissatisfaction primarily due to the fact that a common mechanism of their occurrence has not been understood.

More than fifty years of a study of the B-BCS-theory has shown that this theory successfully describes the general features of the phenomenon, but it can not be developed in the theory of superconductors. It explains general laws such as the emergence of the energy gap, the behavior of specific heat capacity, the flux quantization, etc., but it can not predict the main parameters of the individual superconductors: their critical temperatures and critical magnetic fields. More precisely, in the B-BCS-theory, the expression for the critical temperature of superconductor obtains an exponential form which exponential factor is impossible to measure directly and this formula is of no practical interest.

Recent studies of the isotopic substitution showed that zero-point oscillations of the ions in the metal lattice are not harmonic. Consequently the isotopic substitution affects the interatomic distances in a lattice, and as the result, they directly change the Fermi energy of a metal [50].

Therefore, the assumption developed in the middle of the last century, that the electron-phonon interaction is the only possible mechanism of superconductivity was proved to be wrong. The direct effect of isotopic substitution on the Fermi energy gives a possibility to consider the superconductivity without the phonon mechanism.

Furthermore, a closer look at the problem reveals that the B-BCS-theory describes the mechanism of electron pairing, but in this theory there is no mechanism for combining pairs in the single super-ensemble. The necessary condition for the existence of

superconductivity is formation of a unique ensemble of particles. By this mechanism, a very small amount of electrons are combined in super-ensemble, on the level 10 in minus fifth power from the full number of free electrons. This fact also can not be understood in the framework of the B-BCS theory.

An operation of the mechanism of electron pairing and turning them into boson pairs is a necessary but not sufficient condition for the existence of a superconducting state. Obtained pairs are not identical at any such mechanism. They differ because of their uncorrelated zero-point oscillations and they can not form the condensate at that.

At very low temperatures, that allow superfluidity in helium and superconductivity in metals, all movements of particles are stopped except for their zero-point oscillations. Therefore, as an alternative, we should consider the interaction of super-particles through electro-magnetic fields of zero-point oscillations. This approach was proved to be fruitful. At the consideration of super-phenomena as consequences of the zero-point oscillations ordering, one can construct theoretical mechanisms enabling to give estimations for the critical parameters of these phenomena which are in satisfactory agreement with measurements.

As result, one can see that as the critical temperatures of (type-I) superconductors are equal to about 10^{-6} from the Fermi temperature for superconducting metal, which is consistent with data of measurements. At this the destruction of superconductivity by application of critical magnetic field occurs when the field destroys the coherence of zero-point oscillations of electron pairs. This is in good agreement with measurements also.

A such-like mechanism works in superfluid liquid helium. The problem of the interaction of zero-point oscillations of the electronic shells of neutral atoms in the s-state, was considered yet before the World War II by F. London. He has shown that this interaction is responsible for the liquefaction of helium. The closer analysis of interactions of zero-point oscillations for helium atomic shells shows that at first at the temperature of about 4K only, one of the oscillations mode becomes ordered. As a result, the forces of attraction appear between atoms which are need for helium liquefaction. To create a single quantum ensemble, it is necessary to reach the complete ordering of atomic oscillations. At the complete ordering of oscillations at about 2K, the additional energy of the mutual attraction appears and the system of helium-4 atoms transits in superfluid state. To form the superfluid quantum ensemble in Helium-3, not only the

zero-point oscillations should be ordered, but the magnetic moments of the nuclei should be ordered too. For this reason, it is necessary to lower the temperature below 0.001K. This is also in agreement with experiment.

Thus it is possible to show that both related super-phenomena, superconductivity and superfluidity, are based on the single physical mechanism: the ordering of zero-point oscillations.

The roles of zero-point oscillations in formation of the superconducting state have been previously considered in papers [37]-[39].

6.2   The Electron Pairing

J. Bardeen was first who turned his attention toward a possible link between superconductivity and zero-point oscillations [40]. The special role of zero-point vibrations exists due to the fact that at low temperatures all movements of electrons in metals have been frozen except for these oscillations.

Fermi-energy level for
non-interacting free electrons

The lowering of electron energy at their pairing
due to magnetic dipole-dipole interaction

The lowering of electron energy due to
the ordering of their zero-point oscillations

The condensate of
ordered zero-point
oscillations
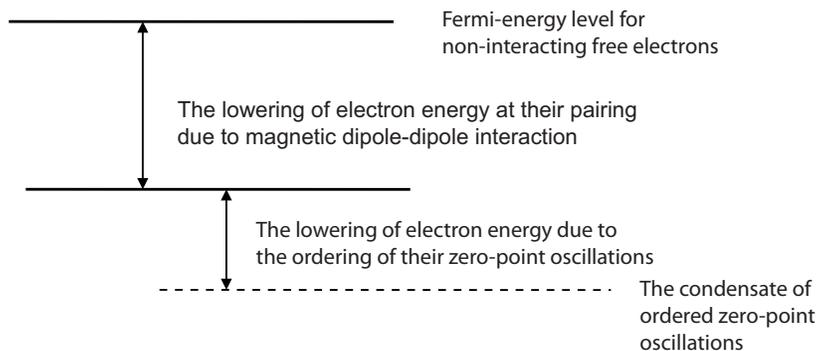
Figure 6.1   *The schematic representation of the energy levels of conducting electrons
in a superconducting metal.*

Superconducting condensate formation requires two mechanisms: first, the electrons must be united in boson pairs, and then the zero-point fluctuations must be ordered (see Figure 6.1).

The energetically favorable pairing of electrons in the electron gas should occur above the critical temperature.

Possibly, the pairing of electrons can occur due to the magnetic dipole-dipole interaction.

For the magnetic dipole-dipole interaction, to merge two electrons into the singlet pair at the temperature of about 10K, the distance between these particles must be small enough:

$$r < (\mu_B^2/kT_c)^{1/3} \approx a_B, \tag{6.1}$$

where $a_B = \frac{\hbar^2}{m_e e^2}$ is the Bohr radius.

That is, two collectivized electrons must be localized in one lattice site volume. It is agreed that the superconductivity can occur only in metals with two collectivized electrons per atom, and cannot exist in the monovalent alkali and noble metals.

It is easy to see that the presence of magnetic moments on ion sites should interfere with the magnetic combination of electrons. This is confirmed by the experimental fact: as there are no strong magnetic substances among superconductors, so adding of iron, for example, to traditional superconducting alloys always leads to a lower critical temperature.

On the other hand, this magnetic coupling should not be destroyed at the critical temperature. The energy of interaction between two electrons, located near one lattice site, can be much greater. This is confirmed by experiments showing that throughout the period of the magnetic flux quantization, there is no change at the transition through the critical temperature of superconductor [41], [42].

The outcomes of these experiments are evidence that the existence of the mechanism of electron pairing is a necessary but not a sufficient condition for the existence of superconductivity.

The magnetic mechanism of electronic pairing proposed above can be seen as an assumption which is consistent with the measurement data and therefore needs a more detailed theoretic consideration and further refinement.

On the other hand, this issue is not very important in the grander scheme, because the

nature of the mechanism that causes electron pairing is not of a significant importance. Instead, it is important that there is a mechanism which converts the electronic gas into an ensemble of charged bosons with zero spin in the considered temperature range (as well as in a some range of temperatures above T_c).

If the temperature is not low enough, the electronic pairs still exist but their zero-point oscillations are disordered. Upon reaching the T_c, the interaction between zero-point oscillations should cause their ordering and therefore a superconducting state is created.

6.3   The Interaction of Zero-Point Oscillations
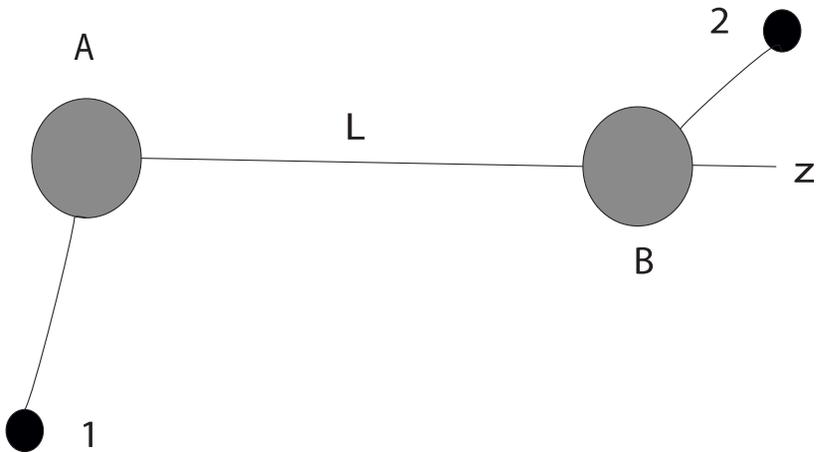
Figure 6.2   *Two ions placed on the distance L and centers of their electronic clouds.*

The principal condition for the superconducting state formation is the ordering of zero-point oscillations. It is realized because the paired electrons obeying Bose-Einstein statistics attract each other.

The origin of this attraction can be explained as follows.

Let two ion A and B be located on the z axis at the distance L from each other. Two collectivized electrons create clouds with centers at points 1 and 2 in the vicinity of each ions (Figure 6.2). Let r_1 be the radius-vector of the center of the first electronic cloud

relative to the ion A and r_2 is the radius-vector of the second electron relative to the ion B.

Following the Born-Oppenheimer approximation, slowly oscillating ions are assumed fixed. Let the temperature be low enough $(T \to 0)$, so only zero-point fluctuations of electrons would be taken into consideration.

In this case, the Hamiltonian of the system can be written as:

$$
\begin{aligned}
&H = H_0 + H' \\
&H_0 = -\frac{\hbar^2}{4m_e}\left(\nabla_1^2 + \nabla_2^2\right) - \frac{4e^2}{r_1} - \frac{4e^2}{r_2} \\
&\\
&H' = \frac{4e^2}{L} + \frac{4e^2}{r_{12}} - \frac{4e^2}{r_{1B}} - \frac{4e^2}{r_{2A}}
\end{aligned}
\tag{6.2}
$$

Eigenfunctions of the unperturbed Hamiltonian describes two ions surrounded by electronic clouds without interactions between them. Due to the fact that the distance between the ions is large compared with the size of the electron clouds $L \gg r$, the additional term H' characterizing the interaction can be regarded as a perturbation.

If we are interested in the leading term of the interaction energy for L, the function H' can be expanded in a series in powers of $1/L$ and we can write the first term:

$$
\begin{aligned}
H' = \frac{4e^2}{L}\Bigg\{ 1 + &\left[1 + \frac{2(z_2 - z_1)}{L} + \frac{(x_2-x_1)^2 + (y_2-y_1)^2 + (z_2-z_1)^2}{L^2} \right]^{-1/2} \\
&- \left(1 - \frac{2z_1}{L} + \frac{r_1^2}{L^2} \right)^{-1/2} - \left(1 + \frac{2z_2}{L} + \frac{r_2^2}{L^2} \right)^{-1/2} \Bigg\}.
\end{aligned}
\tag{6.3}
$$

After combining the terms in this expression, we get:

$$
H' \approx \frac{4e^2}{L^3}\left(x_1 x_2 + y_1 y_2 - 2 z_1 z_2 \right).
\tag{6.4}
$$

This expression describes the interaction of two dipoles d_1 and d_2, which are formed by fixed ions and electronic clouds of the corresponding instantaneous configuration.

Let us determine the displacements of electrons which lead to an attraction in the system .

Let zero-point fluctuations of the dipole moments formed by ions with their electronic clouds occur with the frequency Ω_0, whereas each dipole moment can be decomposed

into three orthogonal projection $d_x = ex, d_y = ey$ and $d_z = ez$, and fluctuations of the second clouds are shifted in phase on φ_x, φ_y and φ_z relative to fluctuations of the first.

As can be seen from Eq.(6.4), the interaction of z-components is advantageous at in-phase zero-point oscillations of clouds, i.e., when $\varphi_z = 2\pi$.

Since the interaction of oscillating electric dipoles is due to the occurrence of oscillating electric field generated by them, the phase shift on 2π means that attracting dipoles are placed along the z-axis on the wavelength Λ_0:

$$L_z = \Lambda_0 = \frac{c}{2\pi\Omega_0}. \tag{6.5}$$

As follows from (6.4), the attraction of dipoles at the interaction of the x and y-component will occur if these oscillations are in antiphase, i.e. if the dipoles are separated along these axes on the distance equals to half of the wavelength:

$$L_{x,y} = \frac{\Lambda_0}{2} = \frac{c}{4\pi\Omega_0}. \tag{6.6}$$

In this case

$$H' = -4e^2 \left(\frac{x_1 x_2}{L_x^3} + \frac{y_1 y_2}{L_y^3} + 2\frac{z_1 z_2}{L_z^3} \right). \tag{6.7}$$

Assuming that the electronic clouds have isotropic oscillations with amplitude a_0 for each axis

$$x_1 = x_2 = y_1 = y_2 = z_1 = z_2 = a_0 \tag{6.8}$$

we obtain

$$H' = 576\pi^3 \frac{e^2}{c^3} \Omega_0^3 a_0^2. \tag{6.9}$$

6.4 The Zero-Point Oscillations Amplitude

The principal condition for the superconducting state formation, that is the ordering of zero-point oscillations, is realized due to the fact that the paired electrons, which obey Bose-Einstein statistics, interact with each other.

At they interact, their amplitudes, frequencies and phases of zero-point oscillations become ordered.

Let an electron gas has density n_e and its Fermi-energy be $\mathcal{E}_F$. Each electron of this gas can be considered as fixed inside a cell with linear dimension λ_F:[1]

$$\lambda_F^3 = \frac{1}{n_e} \tag{6.10}$$

which corresponds to the de Broglie wavelength:

$$\lambda_F = \frac{2\pi\hbar}{p_F}. \tag{6.11}$$

Having taken into account (6.11), the Fermi energy of the electron gas can be written as

$$\mathcal{E}_F = \frac{p_F^2}{2m_e} = 2\pi^2 \frac{e^2 a_B}{\lambda_F^2}. \tag{6.12}$$

However, a free electron interacts with the ion at its zero-point oscillations. If we consider the ions system as a positive background uniformly spread over the cells, the electron inside one cell has the potential energy:

$$\mathcal{E}_p \simeq -\frac{e^2}{\lambda_F}. \tag{6.13}$$

As zero-point oscillations of the electron pair are quantized by definition, their frequency and amplitude are related

$$m_e a_0^2 \Omega_0 \simeq \frac{\hbar}{2}. \tag{6.14}$$

Therefore, the kinetic energy of electron undergoing zero-point oscillations in a limited region of space, can be written as:

$$\mathcal{E}_k \simeq \frac{\hbar^2}{2m_e a_0^2}. \tag{6.15}$$

---

[1] Of course, the electrons are quantum particles and their fixation cannot be considered too literally. Due to the Coulomb forces of ions, it is more favorable for collectivized electrons to be placed near the ions for the shielding of ions fields. At the same time, collectivized electrons are spread over whole metal. It is wrong to think that a particular electron is fixed inside a cell near to a particular ion. But the spread of the electrons does not play a fundamental importance for our further consideration, since there are two electrons near the node of the lattice in the divalent metal at any given time. They can be considered as located inside the cell as averaged.

In accordance with the virial theorem [45], if a particle executes a finite motion, its potential energy $\mathcal{E}_p$ should be associated with its kinetic energy $\mathcal{E}_k$ through the simple relation $|\mathcal{E}_p| = 2\mathcal{E}_k$.

In this regard, we find that the amplitude of the zero-point oscillations of an electron in a cell is:

$$a_0 \simeq \sqrt{2\lambda_F a_B}. \tag{6.16}$$

6.5   The Condensation Temperature

Hence the interaction energy, which unites particles into the condensate of ordered zero-point oscillations

$$\Delta_0 \equiv H' = 18\pi^3 \alpha^3 \frac{e^2 a_B}{\lambda_F^2}, \tag{6.17}$$

where $\alpha = \frac{1}{137}$ is the fine structure constant.

Comparing this association energy with the Fermi energy (6.12), we obtain

$$\frac{\Delta_0}{\mathcal{E}_F} = 9\pi\alpha^3 \simeq 1.1 \cdot 10^{-5}. \tag{6.18}$$

Assuming that the critical temperature below which the possible existence of such condensate is approximately equal

$$T_c \simeq \frac{1}{2}\frac{\Delta_0}{k} \tag{6.19}$$

(the coefficient approximately equal to 1/2 corresponds to the experimental data, discussed below in the section (7.6)).

After substituting obtained parameters, we have

$$T_c \simeq 5.5 \cdot 10^{-6} T_F \tag{6.20}$$

The experimentally measured ratios $\frac{T_c}{T_F}$ for I-type superconductors are given in Table 7.1 and in Figure 7.1.

The straight line on this figure is obtained from Eq.(6.20), which as seen defines an upper limit of critical temperatures of I-type superconductors.

Chapter 7

The Condensate of Zero-Point Oscillations and Type-I Superconductors

7.1 The Critical Temperature of Type-I Superconductors

In order to compare the critical temperature of the condensate of zero-point oscillations with measured critical temperatures of superconductors, at first we should make an estimation on the Fermi energies of superconductors. For this we use the experimental data for the Sommerfeld's constant through which the Fermi energy can be expressed:

$$\gamma = \frac{\pi^2 k^2 n_e}{4\mathcal{E}_F} = \frac{1}{2} \cdot \left(\frac{\pi}{3}\right)^{2/3} \left(\frac{k}{\hbar}\right)^2 m_e n_e^{1/3} \tag{7.1}$$

So on the basis of Eqs.(6.12) and (7.1), we get:

$$kT_F(\gamma) = \frac{p_F^2(\gamma)}{2m_e} \simeq \left(\frac{12}{k^2}\right)^2 \left(\frac{\hbar^2}{2m_e}\right)^3 \gamma^2. \tag{7.2}$$

On base of these calculations we obtain possibility to relate directly the critical

temperature of a superconductor with the experimentally measurable parameter: with its electronic specific heat.

Taking into account Eq.(6.20), we have:

$$\Delta_0 \simeq \Theta \gamma^2, \tag{7.3}$$

where the constant

$$\Theta \simeq 31 \frac{\pi^2}{k} \left[\frac{\alpha \hbar^2}{k m_e} \right]^3 \simeq 6.65 \cdot 10^{-22} \frac{K^4 cm^6}{erg}. \tag{7.4}$$

Table 7.1 *The comparison of the calculated values of superconductors critical temperatures with measured Fermi temperatures.*

superconductor	T_c,K	T_F,K Eq(7.2)	$\frac{T_c}{T_F}$
Cd	0.51	$1.81 \cdot 10^5$	$2.86 \cdot 10^{-6}$
Zn	0.85	$3.30 \cdot 10^5$	$2.58 \cdot 10^{-6}$
Ga	1.09	$1.65 \cdot 10^5$	$6.65 \cdot 10^{-6}$
Tl	2.39	$4.67 \cdot 10^5$	$5.09 \cdot 10^{-6}$
In	3.41	$7.22 \cdot 10^5$	$4.72 \cdot 10^{-6}$
Sn	3.72	$7.33 \cdot 10^5$	$5.08 \cdot 10^{-6}$
Hg	4.15	$1.05 \cdot 10^6$	$3.96 \cdot 10^{-6}$
Pb	7.19	$1.85 \cdot 10^6$	$3.90 \cdot 10^{-6}$

Table 7.2   *The comparison of the calculated values of superconductors critical temperatures with measurement data.*

superconductors	T_c(measur), K	$\gamma, \frac{erg}{cm^3 K^2}$	T_c(calc), K Eq.(7.3)	$\frac{T_c(calc)}{T_c(meas)}$
Cd	0.517	532	0.77	1.49
Zn	0.85	718	1.41	1.65
Ga	1.09	508	0.70	0.65
Tl	2.39	855	1.99	0.84
In	3.41	1062	3.08	0.90
Sn	3.72	1070	3.12	0.84
Hg	4.15	1280	4.48	1.07
Pb	7.19	1699	7.88	1.09
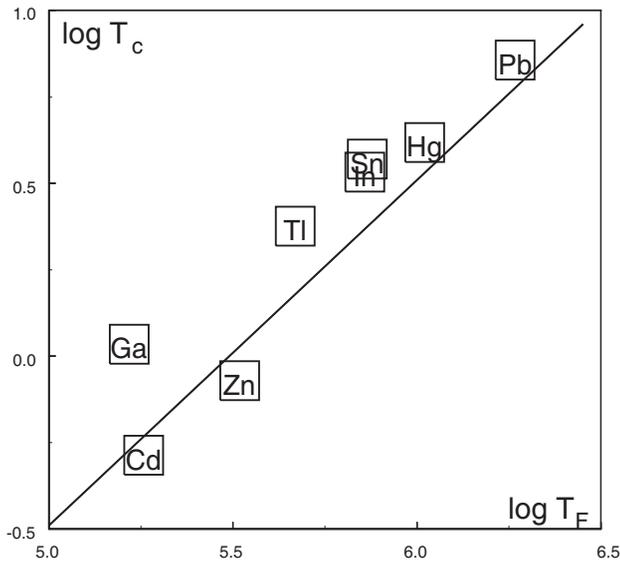
Figure 7.1   *The comparison of critical temperatures T_c of type-I superconductors with their Fermi temperatures T_F. The straight line is obtained from Eq.(6.20).*

The comparison of the calculated parameters and measured data ([22], [32]) is given in Table 7.1-7.2 and in Figure 7.1 and 8.1.

7.2 The Relation of Critical Parameters of Type-I Superconductors

The phenomenon of condensation of zero-point oscillations in the electron gas has its characteristic features.

There are several ways of destroying the zero-point oscillations condensate in electron gas:

Firstly, it can be evaporated by heating. In this case, evaporation of the condensate should possess the properties of an order-disorder transition.

Secondly, due to the fact that the oscillating electrons carry electric charge, the condensate can be destroyed by the application of a sufficiently strong magnetic field.

For this reason, the critical temperature and critical magnetic field of the condensate will be interconnected.

This interconnection should manifest itself through the relationship of the critical temperature and critical field of the superconductors, if superconductivity occurs as result of an ordering of zero-point fluctuations.

Let us assume that at a given temperature $T < T_c$ the system of vibrational levels of conducting electrons consists of only two levels:

- firstly, basic level which is characterized by an anti-phase oscillations of the electron pairs at the distance $\Lambda_0/2$, and

- secondly, an excited level characterized by in-phase oscillation of the pairs.

Let the population of the basic level be N_0 particles and the excited level has N_1 particles.

Two electron pairs at an in-phase oscillations have a high energy of interaction and therefore cannot form the condensate. The condensate can be formed only by the particles that make up the difference between the populations of levels $N_0 - N_1$. In a dimensionless form, this difference defines the order parameter:

$$\Psi = \frac{N_0}{N_0 + N_1} - \frac{N_1}{N_0 + N_1}. \tag{7.5}$$

In the theory of superconductivity, by definition, the order parameter is determined by the value of the energy gap

$$\Psi = \Delta_T / \Delta_0. \tag{7.6}$$

When taking a counting of energy from the level ε_0, we obtain

$$\frac{\Delta_T}{\Delta_0} = \frac{N_0 - N_1}{N_0 + N_1} \simeq \frac{e^{2\Delta_T/kT} - 1}{e^{2\Delta_T/kT} + 1} = th(2\Delta_T/kT). \tag{7.7}$$

Passing to dimensionless variables $\delta \equiv \frac{\Delta_T}{\Delta_0}$, $t \equiv \frac{kT}{kT_c}$ and $\beta \equiv \frac{2\Delta_0}{kT_c}$ we have

$$\delta = \frac{e^{\beta\delta/t} - 1}{e^{\beta\delta/t} + 1} = th(\beta\delta/t). \tag{7.8}$$

This equation describes the temperature dependence of the energy gap in the spectrum of zero-point oscillations. It is similar to other equations describing other physical phenomena, that are also characterized by the existence of the temperature dependence of order parameters [43], [44]. For example, this dependence is similar to temperature dependencies of the concentration of the superfluid component in liquid helium or the spontaneous magnetization of ferromagnetic materials. This equation is the same for all order-disorder transitions (the phase transitions of 2nd-type in the Landau classification).

The solution of this equation, obtained by the iteration method, is shown in Figure 7.2.
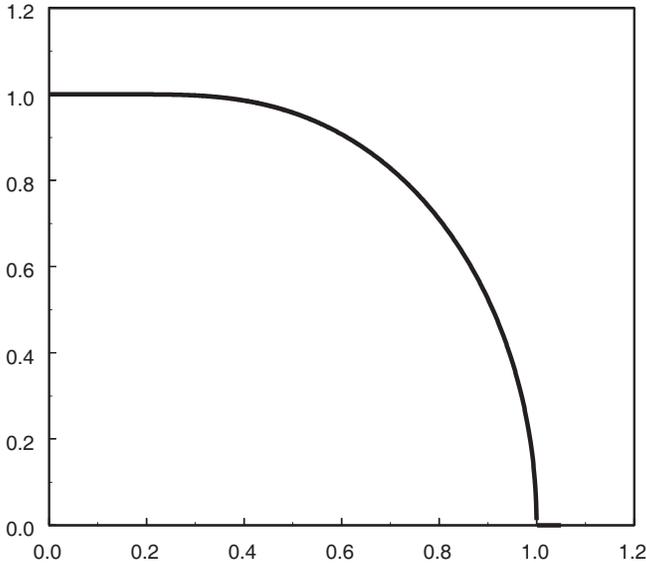
Figure 7.2 *The temperature dependence of the value of the gap in the energetic spectrum of zero-point oscillations calculated on Eq.(7.8).*

This decision is in a agreement with the known transcendental equation of the BCS, which was obtained by the integration of the phonon spectrum, and is in a satisfactory agreement with the measurement data.

After numerical integrating we can obtain the averaging value of the gap:

$$\langle \Delta \rangle = \Delta_0 \int_0^1 \delta dt = 0.852 \, \Delta_0 \, . \tag{7.9}$$

To convert the condensate into the normal state, we must raise half of its particles into the excited state (according to Eq.(7.7), the gap collapses under this condition). To do this, taking into account Eq.(7.9), the unit volume of condensate should have the energy:

$$\mathcal{E}_T \simeq \frac{1}{2} n_0 \langle \Delta_0 \rangle \approx \frac{0.85}{2} \left(\frac{m_e}{2\pi^2 \alpha \hbar^2} \right)^{3/2} \Delta_0^{5/2}, \tag{7.10}$$

On the other hand, we can obtain the normal state of an electrically charged condensate when applying a magnetic field of critical value H_c with the density of energy:

$$\mathcal{E}_H = \frac{H_c^2}{8\pi}. \tag{7.11}$$

As a result, we acquire the condition:

$$\frac{1}{2}n_0\langle\Delta_0\rangle = \frac{H_c^2}{8\pi}. \tag{7.12}$$

This creates a relation of the critical temperature to the critical magnetic field of the zero-point oscillations condensate of the charged bosons.
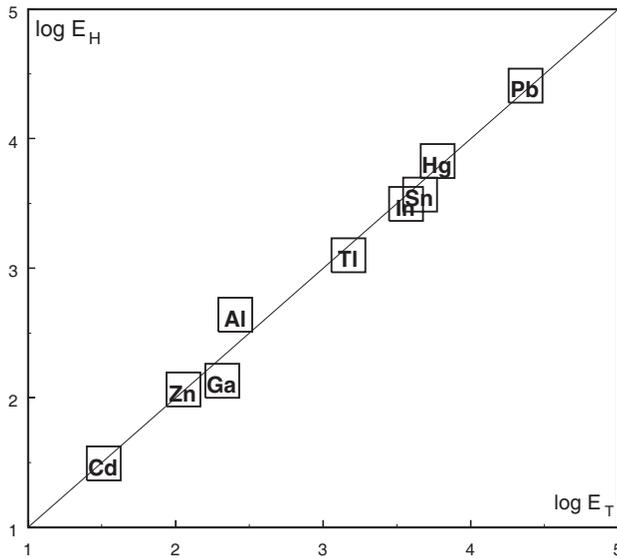
Figure 7.3 *The comparison of the critical energy densities $\mathcal{E}_T$ (Eq.(7.10)) and $\mathcal{E}_H$ (Eq.(7.11)) for the type-I superconductors.*

The comparison of the critical energy densities $\mathcal{E}_T$ and $\mathcal{E}_H$ for type-I superconductors are shown in Figure 7.3.

As shown, the obtained agreement between the energies $\mathcal{E}_T$ (Eq.(7.10)) and $\mathcal{E}_H$ (Eq.(7.11)) is quite satisfactory for type-I superconductors [32], [22]. A similar comparison for type-II superconductors shows results that differ by a factor two approximately. The reason for this will be considered below. The correction of this calculation, has not apparently made sense here. The purpose of these calculations was to show that the description of superconductivity as the effect of the condensation of ordered zero-point oscillations is in accordance with the available experimental data. This goal is considered reached in the simple case of type-I superconductors.

7.3 The Critical Magnetic Field of Superconductors

The direct influence of the external magnetic field of the critical value applied to the electron system is too weak to disrupt the dipole-dipole interaction of two paired electrons:

$$\mu_B H_c \ll k T_c. \tag{7.13}$$

In order to violate the superconductivity, the ordering of the electron zero-point oscillations must be destroyed. For this the presence of relatively weak magnetic field is required.

At combing of Eqs.(7.12), (7.10) and (6.16), we can express the gap through the critical magnetic field and the magnitude of the oscillating dipole moment:

$$\Delta_0 \approx \frac{1}{2} \, e \, a_0 \, H_c. \tag{7.14}$$

The properties of the zero-point oscillations of the electrons should not be dependent on the characteristics of the mechanism of association and also on the condition of the existence of electron pairs. Therefore, we should expect that this equation would also be valid for type-I superconductors, as well as for II-type superconductors (for II-type superconductor $H_c = H_{c1}$ is the first critical field)

An agreement with this condition is illustrated on the Figure 7.4.
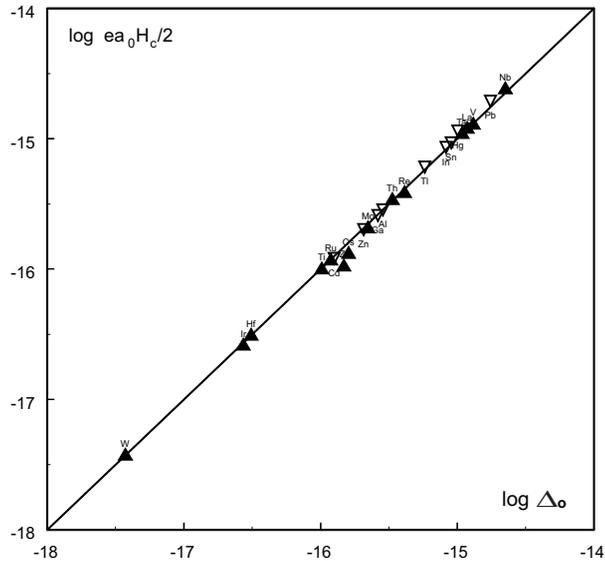
Figure 7.4   *The comparison of the calculated energy of superconducting pairs in the critical magnetic field with the value of the superconducting gap. Here, the following key applies: filled triangles - type-II superconductors, empty triangles - type-I superconductors. On vertical axis - logarithm of the product of the calculated value of the oscillating dipole moment of an electron pair on the critical magnetic field is plotted. On horizontal axis - the value of the gap is shown.*

7.4   The Density of Superconducting Carriers

Let us consider the process of heating the electron gas in metal. When heating, the electrons from levels slightly below the Fermi-energy are raised to higher levels. As a result, the levels closest to the Fermi level, from which at low temperature electrons were forming bosons, become vacant.

At critical temperature T_c, all electrons from the levels of energy bands from $\mathcal{E}_F - \Delta$ to $\mathcal{E}_F$ move to higher levels (and the gap collapses). At this temperature superconductivity is therefore destroyed completely.

This band of energy can be filled by N_Δ particles:

$$N_\Delta = 2 \int_{\mathcal{E}_F - \Delta}^{\mathcal{E}_F} F(\mathcal{E}) D(\mathcal{E}) d\mathcal{E}. \tag{7.15}$$

Where $F(\mathcal{E}) = \frac{1}{e^{\frac{\mathcal{E}-\mu}{\tau}}+1}$ is the Fermi-Dirac function and $D(\mathcal{E})$ is number of states per an unit energy interval, a deuce front of the integral arises from the fact that there are two electron at each energy level.

To find the density of states $D(\mathcal{E})$, one needs to find the difference in energy of the system at $T = 0$ and finite temperature:

$$\Delta \mathcal{E} = \int_0^\infty F(\mathcal{E}) \mathcal{E} D(\mathcal{E}) d\mathcal{E} - \int_0^{\mathcal{E}_F} \mathcal{E} D(\mathcal{E}) d\mathcal{E}. \tag{7.16}$$

For the calculation of the density of states $D(\mathcal{E})$, we must note that two electrons can be placed on each level. Thus, from the expression of the Fermi-energy Eq.(6.12) we obtain

$$D(E_F) = \frac{1}{2} \cdot \frac{dn_e}{d\mathcal{E}_F} = \frac{3n_e}{4\mathcal{E}_F} = \frac{3\gamma}{2k^2\pi^2}, \tag{7.17}$$

where

$$\gamma = \frac{\pi^2 k^2 n_e}{4\mathcal{E}_F} = \frac{1}{2} \cdot \left(\frac{\pi}{3}\right)^{3/2} \left(\frac{k}{\hbar}\right)^2 m_e n_e^{1/3} \tag{7.18}$$

is the Sommerfeld constant [1].

Using similar arguments, we can calculate the number of electrons, which populate the levels in the range from $\mathcal{E}_F - \Delta$ to $\mathcal{E}_F$. For an unit volume of material, Eq.(7.15) can be rewritten as:

$$n_\Delta = 2kT \cdot D(\mathcal{E}_F) \int_{-\frac{\Delta_0}{kT_c}}^0 \frac{dx}{(e^x + 1)}. \tag{7.19}$$

By supposing that for superconductors $\frac{\Delta_0}{kT_c} = 1.86$, as a result of numerical integration we obtain

$$\int_{-\frac{\Delta_0}{kT_c}}^0 \frac{dx}{(e^x + 1)} = [x - ln(e^x + 1)]_{-1.86}^0 \approx 1.22. \tag{7.20}$$

---

[1] It should be noted that because on each level two electrons can be placed, the expression for the Sommerfeld constant Eq.(7.18) contains the additional factor $1/2$ in comparison with the usual formula in literature [44].

Thus, the density of electrons, which throw up above the Fermi level in a metal at temperature $T = T_c$ is

$$n_e(T_c) \approx 2.44 \left(\frac{3\gamma}{k^2 \pi^2} \right) kT_c. \tag{7.21}$$

Where the Sommerfeld constant γ is related to the volume unit of the metal.

From Eq.(6.6) it follows

$$L_0 \simeq \frac{\lambda_F}{\pi \alpha} \tag{7.22}$$

and this forms the ratio of the condensate particle density to the Fermi gas density:

$$\frac{n_0}{n_e} = \frac{\lambda_F^3}{L_0^3} \simeq (\pi \alpha)^3 \simeq 10^{-5}. \tag{7.23}$$

When using these equations, we can find a linear dimension of localization for an electron pair:

$$L_0 = \frac{\Lambda_0}{2} \simeq \frac{1}{\pi \alpha (n_e)^{1/3}}. \tag{7.24}$$

or, taking into account Eq.(6.16), we can obtain the relation between the density of particles in the condensate and the value of the energy gap:

$$\Delta_0 \simeq 2\pi^2 \alpha \frac{\hbar^2}{m_e} n_0^{2/3} \tag{7.25}$$

or

$$n_0 = \frac{1}{L_0^3} = \left(\frac{m_e}{2\pi^2 \alpha \hbar^2} \Delta_0 \right)^{3/2}. \tag{7.26}$$

It should be noted that the obtained ratios for the zero-point oscillations condensate (of bose-particles) differ from the corresponding expressions for the bose-condensate of particles, which can be obtained in many courses (see eg [43]). The expressions for the ordered condensate of zero-point oscillations have an additional coefficient α on the right side of Eq.(7.25).

The de Broglie wavelengths of Fermi electrons expressed through the Sommerfelds constant

$$\lambda_F = \frac{2\pi\hbar}{p_F(\gamma)} \simeq \frac{\pi}{3} \cdot \frac{k^2 m_e}{\hbar^2 \gamma} \tag{7.27}$$

are shown in Table 7.4.

In accordance with Eq.(7.22), which was obtained at the zero-point oscillations consideration, the ratio $\frac{\lambda_F}{\Lambda_0} \simeq 2.3 \cdot 10^{-2}$.

In connection with this ratio, the calculated ratio of the zero-point oscillations condensate density to the density of fermions in accordance with Eq.(7.23) should be near to 10^{-5}.

It can be therefore be seen, that calculated estimations of the condensate parameters are in satisfactory agreement with experimental data of superconductors.

Table 7.3 *The ratios $\frac{\lambda_F}{\Lambda_0}$ and $\frac{n_0}{n_e}$ for type-I superconductors.*

superconductor	λ_F, cm Eq(7.27)	Λ_0,cm Eq(6.6)	$\frac{\lambda_F}{\Lambda_0}$	$\frac{n_0}{n_e} = \left(\frac{\lambda_F}{\Lambda_0}\right)^3$
Cd	$3.1 \cdot 10^{-8}$	$1.18 \cdot 10^{-6}$	$2.6 \cdot 10^{-2}$	$1.8 \cdot 10^{-5}$
Zn	$2.3 \cdot 10^{-8}$	$0.92 \cdot 10^{-6}$	$2.5 \cdot 10^{-2}$	$1.5 \cdot 10^{-5}$
Ga	$3.2 \cdot 10^{-8}$	$0.81 \cdot 10^{-6}$	$3.9 \cdot 10^{-2}$	$6.3 \cdot 10^{-5}$
Tl	$1.9 \cdot 10^{-8}$	$0.55 \cdot 10^{-6}$	$3.4 \cdot 10^{-2}$	$4.3 \cdot 10^{-5}$
In	$1.5 \cdot 10^{-8}$	$0.46 \cdot 10^{-6}$	$3.2 \cdot 10^{-2}$	$3.8 \cdot 10^{-5}$
Sn	$1.5 \cdot 10^{-8}$	$0.44 \cdot 10^{-6}$	$3.4 \cdot 10^{-2}$	$4.3 \cdot 10^{-5}$
Hg	$1.3 \cdot 10^{-8}$	$0.42 \cdot 10^{-6}$	$3.1 \cdot 10^{-2}$	$2.9 \cdot 10^{-5}$
Pb	$1.0 \cdot 10^{-8}$	$0.32 \cdot 10^{-6}$	$3.1 \cdot 10^{-2}$	$2.9 \cdot 10^{-5}$

Table 7.4   *The comparison of the superconducting carriers density at $T = 0$ with the density of thermally activated electrons at $T = T_c$.*

superconductor	n_0	$n_e(T_c)$	$2n_0/n_e(T_c)$
Cd	$6.11 \cdot 10^{17}$	$1.48 \cdot 10^{18}$	0.83
Zn	$1.29 \cdot 10^{18}$	$3.28 \cdot 10^{18}$	0.78
Ga	$1.85 \cdot 10^{18}$	$2.96 \cdot 10^{18}$	1.25
Al	$2.09 \cdot 10^{18}$	$8.53 \cdot 10^{18}$	0.49
Tl	$6.03 \cdot 10^{18}$	$1.09 \cdot 10^{19}$	1.10
In	$1.03 \cdot 10^{19}$	$1.94 \cdot 10^{19}$	1.06
Sn	$1.18 \cdot 10^{19}$	$2.14 \cdot 10^{19}$	1.10
Hg	$1.39 \cdot 10^{19}$	$2.86 \cdot 10^{19}$	0.97
Pb	$3.17 \cdot 10^{19}$	$6.58 \cdot 10^{19}$	0.96

Based on these calculations, it is interesting to compare the density of superconducting carriers n_0 at $T = 0$, which is described by Eq.(7.26), with the density of normal carriers $n_e(T_c)$, which are evaporated on levels above $\mathcal{E}_F$ at $T = T_c$ and are described by Eq.(7.21).
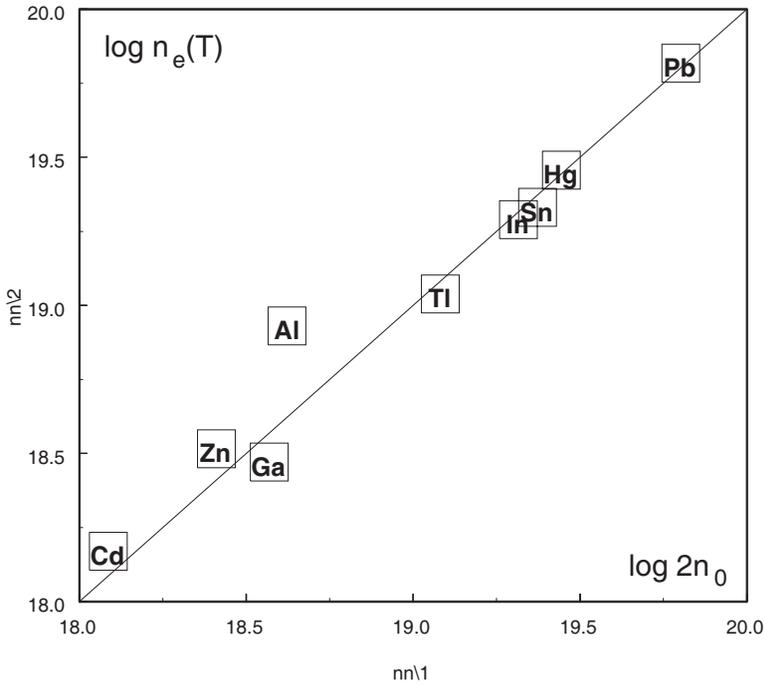
Figure 7.5 *The comparison of the number of superconducting carriers at $T = 0$ with the number of thermally activated electrons at $T = T_c$.*

This comparison is shown in Table 7.4 and Figure 7.5. (Data has been taken from the tables [32], [22]).

From the data described above, we can obtain the condition of destruction of superconductivity, after heating for superconductors of type-I, as written in the equation:

$$n_e(T_c) \simeq 2n_0 \tag{7.28}$$

7.5 The Sound Velocity of the Zero-Point Oscillations Condensate

The wavelength of zero-point oscillations Λ_0 in this model is an analogue of the Pippard coherence length in the BCS. As usually accepted [22], the coherence length $\xi = \frac{\hbar v_F}{4\Delta_0}$.

The ratio of these lengths, taking into account Eq.(6.20), is simply the constant:

$$\frac{\Lambda_0}{\xi} \approx 8\pi^2\alpha^2 \approx \cdot 10^{-3}. \tag{7.29}$$

The attractive forces arising between the dipoles located at a distance $\frac{\Lambda_0}{2}$ from each other and vibrating in opposite phase, create pressure in the system:

$$P \simeq \frac{d\Delta_0}{dV} \simeq \frac{d_\Omega^2}{L_0^6}. \tag{7.30}$$

In this regard, sound into this condensation should propagate with the velocity:

$$c_s \simeq \sqrt{\frac{1}{2m_e}\frac{dP}{dn_0}}. \tag{7.31}$$

After the appropriate substitutions, the speed of sound in the condensate can be expressed through the Fermi velocity of electron gas

$$c_s \simeq \sqrt{2\pi^2\alpha^3}v_F \simeq 10^{-2}v_F. \tag{7.32}$$

The condensate particles moving with velocity c_S have the kinetic energy:

$$2m_e c_s^2 \simeq \Delta_0. \tag{7.33}$$

Therefore, by either heating the condensate to the critical temperature when each of its volume obtains the energy $\mathcal{E} \approx n_0\Delta_0$, or initiating the current of its particles with a velocity exceeding c_S, can achieve the destruction of the condensate. (Because the condensate of charged particles oscillations is considered, destroying its coherence can be also obtained at the application of a sufficiently strong magnetic field. See below.)

7.6   The Relationship Δ_0/kT_c

From Eq.(7.28) and taking into account Eqs.(7.3),(7.21) and (7.26), which were obtained for condensate, we have:

$$\frac{\Delta_0}{kT_c} \simeq 1.86. \tag{7.34}$$

This estimation of the relationship Δ_0/kT_c obtained for condensate has a satisfactory agreement with the measured data [32].[2]

Table 7.5 *The value of ratio Δ_0/kT_c obtained experimentally for type-I superconductors.*

superconductor	T_c,K	Δ_0,mev	$\frac{\Delta_0}{kT_c}$
Cd	0.51	0.072	1.64
Zn	0.85	0.13	1.77
Ga	1.09	0.169	1.80
Tl	2.39	0.369	1.79
In	3.41	0.541	1.84
Sn	3.72	0.593	1.85
Hg	4.15	0.824	2.29
Pb	7.19	1.38	2.22

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[2] In the BCS-theory $\frac{\Delta_0}{kT_c} \simeq 1.76$.

Chapter 8

Another Superconductors

8.1   About Type-II Superconductors

In the case of type-II superconductors the situation is more complicated.

In this case, measurements show that these metals have an electronic specific heat that has an order of value greater than those calculated on the base of free electron gas model.

The peculiarity of these metals is associated with the specific structure of their ions. They are transition metals with unfilled inner d-shell (see Table 8.1).

It can be assumed that the increase in the electronic specific heat of these metals should be associated with a characteristic interaction of free electrons with the electrons of the unfilled d-shell.

Since the heat capacity of the ionic lattice of metals is negligible at low temperatures, only the electronic subsystem is thermally active.

Table 8.1 *The external electron shells of elementary type-II superconductors.*

superconductors	electron shells
Ti	$3d^2\, 4s^2$
V	$3d^3\, 4s^2$
Zr	$4d^2\, 5s^2$
Nb	$4d^3\, 5s^2$
Mo	$4d^4\, 5s^2$
Tc	$4d^5\, 5s^2$
Ru	$4d^6\, 5s^2$
La	$5d^1\, 6s^2$
Hf	$5d^2\, 6s^2$
Ta	$5d^3\, 6s^2$
W	$5d^4\, 6s^2$
Re	$5d^5\, 6s^2$
Os	$5d^6\, 6s^2$
Ir	$5d^7\, 6s^2$

At $T = 0$ the superconducting careers populates the energetic level $\mathcal{E}_F - \Delta_0$. During the destruction of superconductivity through heating, an each heated career increases its thermal vibration. If the effective velocity of vibration is v_t, its kinetic energy:

$$\mathcal{E}_k = \frac{mv_t^2}{2} \simeq \Delta_0 \tag{8.1}$$

Only a fraction of the heat energy transferred to the metal is consumed in order to increase the kinetic energy of the electron gas in the transition metals.

Another part of the energy will be spent on the magnetic interaction of a moving electron.

At contact with the d-shell electron, a freely moving electron induces onto it the magnetic field of the order of value:

$$H \approx \frac{e}{r_c^2} \frac{v}{c}. \tag{8.2}$$

The magnetic moment of d-electron is approximately equal to the Bohr magneton. Therefore the energy of the magnetic interaction between a moving electron of conductivity and a d-electron is approximately equal to:

$$\mathcal{E}_\mu \approx \frac{e^2}{2r_c} \frac{v}{c}. \tag{8.3}$$

This energy is not connected with the process of destruction of superconductivity.

Whereas, in metals with a filled d-shell (type-I superconductors), the whole heating energy increases the kinetic energy of the conductivity electrons and only a small part of the heating energy is spent on it in transition metals:

$$\frac{\mathcal{E}_k}{\mathcal{E}_\mu + \mathcal{E}_k} \simeq \frac{mv_t}{h} a_B. \tag{8.4}$$

So approximately

$$\frac{\mathcal{E}_k}{\mathcal{E}_\mu + \mathcal{E}_k} \simeq \frac{a_B}{L_0}. \tag{8.5}$$

Therefore, whereas the dependence of the gap in type-I superconductors from the heat capacity is defined by Eq.(7.3), it is necessary to take into account the relation Eq.(8.5) in type-II superconductors for the determination of this gap dependence. As a result of this estimation, we can obtain:

$$\Delta_0 \simeq \Theta \gamma^2 \left(\frac{\mathcal{E}_k}{\mathcal{E}_\mu + \mathcal{E}_k} \right) \simeq \Theta \gamma^2 \left(\frac{a_B}{L_0} \right) \frac{1}{2}, \tag{8.6}$$

where $1/2$ is the fitting parameter.
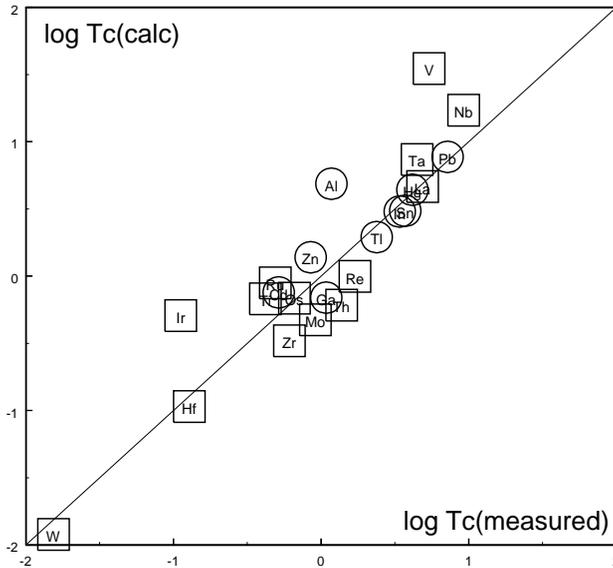
Figure 8.1 *The comparison of the calculated values of critical temperatures of superconductors with measurement data. Circles relate to type-I superconductors, squares show type-II superconductors. On the abscissa, the measured values of critical temperatures are plotted, on ordinate, the calculated estimations are plotted. The calculations of critical temperatures for type-I superconductors were made by using Eq.(7.3) and the estimations for type-II superconductors was obtained by using Eq.(8.6).*

The comparison of the results of these calculations with the measurement data (Figure 8.1) shows that for the majority of type-II superconductors the estimation Eq.(8.6) can be considered quite satisfactory.[1]

---

[1] The lowest critical temperature was measured for Mg. It is approximately equal to 1mK. Mg-atoms in the metallic state are given two electrons into the electron gas of conductivity. It is confirmed by the fact that the pairing of these electrons, which manifests itself in the measured value of the flux quantum [42], is observed above T_c. It would seem that in view of this metallic Mg-ion must have electron shell like the Ne-atom. Therefore it is logical to expect that the critical temperature of Mg can be calculated by the formula for I-type superconductors. But actually in order to get the value of $T_c \approx 1mK$, the critical temperature of Mg should be calculated by the formula (8.6), which is applicable to the description of metals with an unfilled inner shell. This suggests that the ionic core of magnesium metal apparently is not as simple as the completely filled Ne-shell.

8.2   Alloys and High-Temperature Superconductors

In order to understand the mechanism of high temperature superconductivity, it is important to establish whether the high-T_c ceramics are the I or II-type superconductors, or whether they are a special class of superconductors.

In order to determine this, we need to look at the above established dependence of critical parameters from the electronic specific heat and also consider that the specific heat of superconductors I and II-types are differing considerably.

There are some difficulties by determining the answer this way: as we do not precisely know the density of the electron gas in high-temperature superconductors. However, the densities of atoms in metals do not differ too much and we can use Eq.(7.3) for the solution of the problem of the I- and II-types superconductors distinguishing.

If parameters of type-I superconductors are inserted into this equation, we obtain quite a satisfactory estimation of the critical temperature (as was done above, see Figure 8.1). For the type-II superconductors' values, this assessment gives an overestimated value due to the fact that type-II superconductors' specific heat has additional term associated with the magnetization of d-electrons.

This analysis therefore, illustrates a possibility where we can divide all superconductors into two groups, as is evident from the Figure 8.2.

It is generally assumed that we consider alloys Nb_3Sn and V_3Si as the type-II superconductors. This assumption seems quite normal because they are placed in close surroundings of Nb.   Some excess of the calculated critical temperature over the experimentally measured value for ceramics $Ta_2Ba_2Ca_2Cu_3O_{10}$ can be attributed to the measured heat capacity that may have been created by not only conductive electrons, but also non-superconducting elements (layers) of ceramics. It is already known that it, as well as ceramics $YBa_2Cu_3O_7$, belongs to the type-II superconductors. However, ceramics (LaSr)$_2$Cu$_4$, Bi-2212 and Tl-2201, according to this figure should be regarded as type-I superconductors, which is unusual.
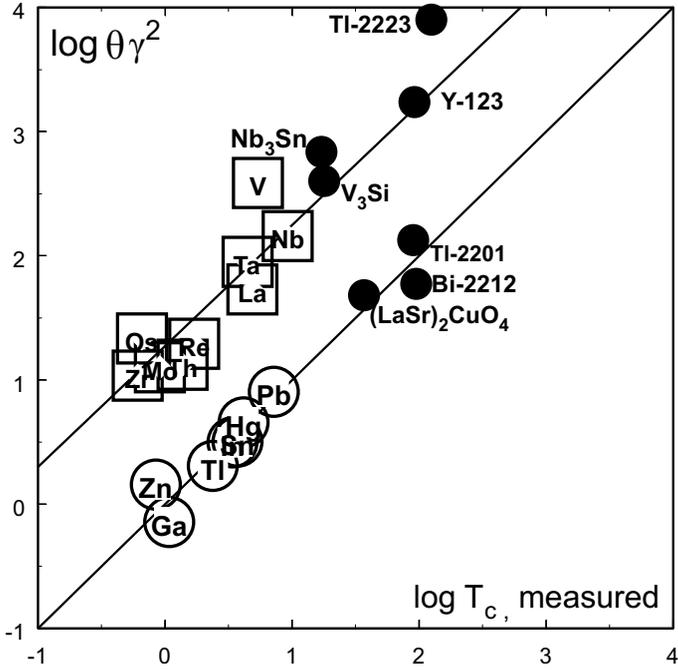
Figure 8.2 *The comparison of the calculated parameter $\Theta\gamma^2$ with the measurement of the critical temperatures of elementary superconductors and some superconducting compounds.*

Chapter 9

About the London Penetration Depth

9.1 The Magnetic Energy of a Moving Electron

To avoid these incorrect results, let us consider a balance of magnetic energy in a superconductor within magnetic field. This magnetic energy is composed of energy from a penetrating external magnetic field and magnetic energy of moving electrons.

By using formulas [46], let us estimate the ratio of the magnetic and kinetic energy of an electron (the charge of e and the mass m_e) when it moves rectilinearly with a velocity $v \ll c$.

The density of the electromagnetic field momentum is expressed by the equation:

$$\mathbf{g} = \frac{1}{4\pi c}[\mathbf{EH}] \tag{9.1}$$

While moving with a velocity $\mathbf{v}$, the electric charge carrying the electric field with intensity E creates a magnetic field

$$\mathbf{H} = \frac{1}{c}[\mathbf{Ev}] \tag{9.2}$$

with the density of the electromagnetic field momentum (at $v \ll c$)

$$\mathbf{g} = \frac{1}{4\pi c^2}[\mathbf{E}[\mathbf{v}\mathbf{E}]] = \frac{1}{4\pi c^2}\left(\mathbf{v}E^2 - \mathbf{E}(\mathbf{v} \cdot \mathbf{E})\right) \tag{9.3}$$

As a result, the momentum of the electromagnetic field of a moving electron

$$\mathbf{G} = \int_V \mathbf{g}dV = \frac{1}{4\pi c^2}\left(\mathbf{v}\int_V E^2\,dV - \int_V \mathbf{E}\,E\,v\,cos\vartheta\,dV\right) \tag{9.4}$$

The integrals are taken over the entire space, which is occupied by particle fields, and ϑ is the angle between the particle velocity and the radius vector of the observation point. By calculating the last integral in the condition of the axial symmetry with respect to $\mathbf{v}$, the contributions from the components of the vector $\mathbf{E}$, which is perpendicular to the velocity, cancel each other for all pairs of elements of the space (if they located diametrically opposite on the magnetic force line). Therefore, according to Eq.(9.4), the component of the field which is collinear to $\mathbf{v}$

$$\frac{E\,cos\vartheta \cdot \mathbf{v}}{v} \tag{9.5}$$

can be taken instead of the vector $\mathbf{E}$. By taking this information into account, going over to the spherical coordinates and integrating over angles, we can obtain

$$\mathbf{G} = \frac{\mathbf{v}}{4\pi c^2}\int_r^\infty E^2 \cdot 4\pi r^2\,dr \tag{9.6}$$

If we limit the integration of the field by the Compton electron radius $r_C = \frac{\hbar}{m_e c}$, [1] then $v \ll c$, and we obtain:

$$\mathbf{G} = \frac{\mathbf{v}}{4\pi c^2}\int_{r_C}^\infty E^2 \cdot 4\pi r^2\,dr = \frac{\mathbf{v}}{c^2}\frac{e^2}{r_C}. \tag{9.7}$$

In this case by taking into account Eq.(9.2), the magnetic energy of a slowly moving electron pair is equal to:

$$\mathcal{E} = \frac{vG}{2} = \frac{v^2}{c^2}\frac{e^2}{2r_C} = \alpha\frac{m_e v^2}{2}. \tag{9.8}$$

---

[1]   Such effects as the pair generation force us to consider the radius of the "quantum electron" as approximately equal to Compton radius [47].

9.2   The Magnetic Energy and the London Penetration Depth

The energy of external magnetic field into volume dv:

$$\mathcal{E} = \frac{H^2}{8\pi} dv. \tag{9.9}$$

At a density of superconducting carriers n_s, their magnetic energy per unit volume in accordance with (9.8):

$$\mathcal{E}_H \simeq \alpha n_s \frac{m_2 v^2}{2} = \alpha \frac{m_e j_s^2}{2n_s e}, \tag{9.10}$$

where $j_s = 2 e n_s v_s$ is the density of a current of superconducting carriers.

Taking into account the Maxwell equation

$$\mathbf{rotH} = \frac{4\pi}{c} \mathbf{j}_s, \tag{9.11}$$

the magnetic energy of moving carriers can be written as

$$\mathcal{E}_H \simeq \frac{\widetilde{\Lambda}^2}{8\pi} (rotH)^2, \tag{9.12}$$

where we introduce the notation

$$\widetilde{\Lambda} = \sqrt{\alpha \frac{m_e c^2}{4\pi n_s e^2}} = \sqrt{\alpha} \Lambda_L. \tag{9.13}$$

In this case, part of the free energy of the superconductor connected with the application of a magnetic field is equal to:

$$\mathcal{F}_H = \frac{1}{8\pi} \int_V \left(H^2 + \widetilde{\Lambda}^2 (rotH)^2 \right) dv. \tag{9.14}$$

At the minimization of the free energy, after some simple transformations we obtain

$$\mathbf{H} + \widetilde{\Lambda}^2 \mathbf{rotrotH} = 0, \tag{9.15}$$

thus $\widetilde{\Lambda}$ is the depth of magnetic field penetration into the superconductor.

In view of Eq.(7.26) from Eq.(9.13) we can estimate the values of London penetration depth (see Table 9.2). The consent of the obtained values with the measurement data can be considered quite satisfactory.

Table 9.1   *Corrected values of London penetration depth.*

superconductors	$\lambda_L, 10^{-6}$cm measured [23]	$\widetilde{\Lambda}, 10^{-6}$cm calculated Eq.(9.13)	$\widetilde{\Lambda}/\lambda_L$
Tl	9.2	11.0	1.2
In	6.4	8.4	1.3
Sn	5.1	7.9	1.5
Hg	4.2	7.2	1.7
Pb	3.9	4.8	1.2

The resulting refinement may be important for estimates within the frame of Ginzburg-Landau theory, where the London penetration depth is used as a comparison of calculations and specific parameters of superconductors.

Chapter 10

Three Words to Experimenters

The history of the Medes is obscure and incomprehensible.

Scientists divide it, however, into three periods:

The first is the period, which is absolutely unknown.

The second is one which is followed after the first.

And finally, the third period is a period which is known

to the same degree as two firsts.

A. Averchenko ≪The World History≫

10.1 Why Creation of Room-Temperature Superconductors are Hardly Probably

The understanding of the mechanism of the superconducting state should open a way towards finding a solution to the technological problem. This problem was just a dream

in the last century: the dream to create a superconductor that would be easily produced (in the sense of ductility) and had high critical temperature.

In order to move towards achieving this goal, it is important firstly to understand the mechanism that limits the critical properties of superconductors.

Let us consider a superconductor with a large limiting current. The length of their localisation determines the limiting momentum of superconducting carriers:

$$p_c \simeq \frac{2\pi\hbar}{L_0}. \tag{10.1}$$

Therefore, by using Eq.(7.33), we can compare the critical velocity of superconducting carriers with the sound velocity:

$$v_c = \frac{p_c}{2m_e} \simeq c_s \tag{10.2}$$

and both these velocities are about a hundred times smaller than the Fermi velocity.

The sound velocity in the crystal lattice of metal v_s, in accordance with the Bohm-Staver relation [52], has approximately the same value:

$$v_s \simeq \frac{kT_D}{E_F} v_F \simeq 10^{-2} v_F. \tag{10.3}$$

This therefore, makes it possible to consider superconductivity being destroyed as a superconducting carrier overcomes the sound barrier. That is, if they moved without friction at a speed that was less than that of sound, after it gained speed and the speed of sound was surpassed, it then acquire a mechanism of friction.

Therefore, it is conceivable that if the speed of sound in the metal lattice $v_s < c_s$, then it would create a restriction on the limiting current in superconductor.

If this is correct, then superconductors with high critical parameters should have not only a high Fermi energy of their electron gas, but also a high speed of sound in their lattice.

It is in agreement with the fact that ceramics have higher elastic moduli compared to metals and alloys, and also posses much higher critical temperatures (Figure 10.1).
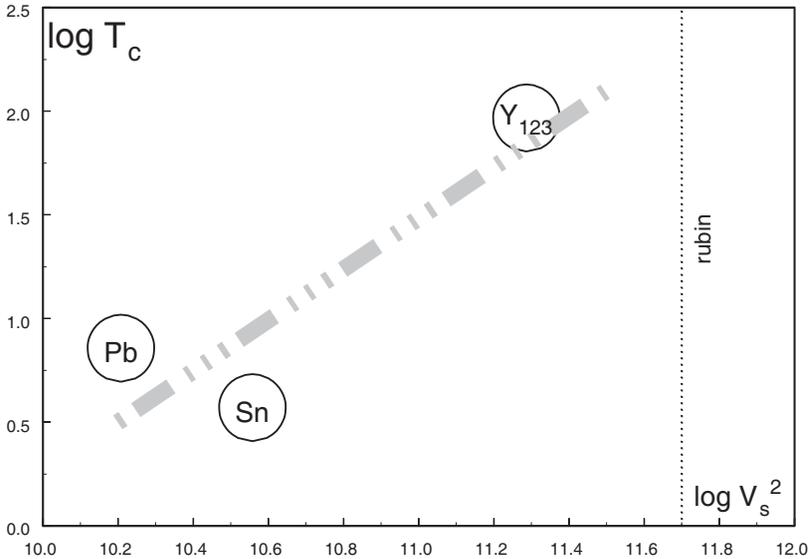
Figure 10.1   *The schematic representation of the dependence of critical temperature on the speed of sound in superconductors. On the ordinate, the logarithm of the superconductor's critical temperature is shown. On the abscissa, the logarithm of the square of the speed of sound is shown (for Sn and Pb - the transverse velocity of sound is shown, because it is smaller). The speed of sound in a film was used for yttrium-123 ceramics. The dashed line shows the value of the transverse velocity of sound in sapphire, as some estimation of the limit of its value. It can be seen that this estimation leads to the restriction on the critical temperature in the range about $0^{o}C$ - the dot-dashed line.*

The dependence of the critical temperature on the square of the speed of sound [48] is illustrated in Figure 10.1.

This figure, which can be viewed only as a rough estimation due to the lack of necessary experimental data, shows that the elastic modulus of ceramics with a critical temperature close to the room temperature, should be close to the elastic modulus of sapphire, which is very difficult to achieve.

In addition, such ceramics would be deprived of yet another important quality - their adaptability. Indeed, in order to obtain a thin wire, we require a plastic superconductor.

A solution of this problem would be to find a material that possesses an acceptably high critical temperature (above 80K) and also experiences a phase transition at an even higher temperature of heat treatment. It would be possible to make a thin wire from a superconductor near the point of phase transition, as the elastic modules are typically not usually very strong at this stage.

10.2 Magnetic Electron Pairing

This considered formation of mechanism for the superconducting state provides a possibility of obtaining the estimations of the critical parameters of superconductors, which in most cases is in satisfactory agreement with measured data. For some superconductors, this agreement is stronger, and for other, such as Ir, Al, V (see Figure 8.1), it is expedient to carry out further theoretical and experimental studies due to causes of deviations.

The mechanism of magnetic electron pairing is also of fundamental interest in order to further clarify this.

As was found earlier, in the cylinders made from certain superconducting metals (Al[41] and Mg[42]), the observed magnetic flux quantization has exactly the same period above T_c and that below T_c. The authors of these studies attributed this to the influence of a special effect. It seems more natural to think that the stability of the period is a result of the pairing of electrons due to magnetic dipole-dipole interaction continuing to exist at temperatures above T_c, despite the disappearance of the material's superconducting properties. At this temperature the coherence of the zero-point fluctuations is destroyed, and with it so is the superconductivity.

The pairing of electrons due to dipole-dipole interaction should be absent in the monovalent metals. In these metals, the conduction electrons are localized in the lattice at very large distances from each other.

It is therefore interesting to compare the period of quantization in the two cases. In a thin cylinder made of a superconductor, such as Mg, above T_c the quantization period is equal to $\frac{2\pi\hbar c}{2e}$. In the same cylinder of a noble metal (such as gold), the sampling period

should be twice as large.

10.3   The Effect of Isotopic Substitution on the Condensation of Zero-Point Oscillations

The attention of experimentalists could be attracted to the isotope effect in superconductors, which served as a starting point of the B-BCS theory. In the 50's, it had been experimentally established that there is a dependence of the critical temperature of superconductors due to the mass of the isotope. As the effect depends on the ionic mass, this is considered to be due to the fact that it is based on the vibrational (phonon) process.

The isotope effect for a number of I-type superconductors - Zn, Sn, In, Hg, Pb - can be described by the relationship:

$$\sqrt{M_i}T_c = const, \tag{10.4}$$

where M_i is the mass of the isotope, T_c is the critical temperature. The isotope effect in other superconductors can either be described by other dependencies, or is absent altogether.

In recent decades, however, the effects associated with the replacement of isotopes in the metal lattice have been studied in detail. It was shown that the zero-point oscillations of ions in the lattice of many metals are non-harmonic. Therefore, the isotopic substitution can directly affect the lattice parameters, the density of the lattice and the density of the electron gas in the metal, on its Fermi energy and on other properties of the electronic subsystem.

The direct study of the effect of isotopic substitution on the lattice parameters of superconducting metals has not been carried out.

The results of measurements made on Ge, Si, diamond and light metals, such as Li [50], [49] (researchers prefer to study crystals, where the isotope effects are large, and it is easier to carry out appropriate measurements), show that there is square-root dependence of the force constants on the isotope mass, which was required by Eq.(10.4). The same dependence of the force constants on the mass of the isotope has been found in tin [51].

Unfortunately, no direct experiments of the effect of isotopic substitution on the electronic properties (such as the electronic specific heat and the Fermi energy), exist for metals substantial for our consideration.

Let us consider what should be expected in such measurements. A convenient choice for the superconductor is mercury, as it has many isotopes and their isotope effect has been carefully measured back in the 50s of the last century as aforementioned.

The linear dependence of the critical temperature of a superconductor on its Fermi energy (Eq.(6.20)) and also the existence of the isotope effect suggests the dependence of the ion density in the crystal lattice from the mass of the isotope. Let us consider what should be expected in such measurements.

Even then, it was found that the isotope effect is described by Eq.(10.4) in only a few superconductors. In others, it displays different values, and therefore in a general case it can be described by introducing of the parameter $\mathfrak{a}$:

$$M_i^{\mathfrak{a}} T_c = Const. \tag{10.5}$$

At taking into account Eq.(6.20), we can write

$$T_c \sim \mathcal{E}_F \sim n_e^{2/3} \tag{10.6}$$

The parameter l which characterizes the ion lattice obtains an increment Δl with an isotope substitution:

$$\frac{\Delta l}{l} = -\frac{\mathfrak{a}}{2} \cdot \frac{\Delta M_i}{M_i}, \tag{10.7}$$

where M_i and ΔM_i are the mass of isotope and its increment.

It is generally accepted that in an accordance with the terms of the phonon mechanism, the parameter $\mathfrak{a} \approx \frac{1}{2}$ for mercury. However, the analysis of experimental data [35]-[36] (see Figure 4.3) shows that this parameter is actually closer to $1/3$. Accordingly, one can expect that the ratio of the mercury parameters is close to:

$$\frac{\left(\frac{\Delta l}{l}\right)}{\left(\frac{\Delta M_i}{M_i}\right)} \approx -\frac{1}{6}. \tag{10.8}$$

Chapter 11

Superfluidity as a Subsequence of Ordering of Zero-Point Oscillations

11.1 Zero-Point Oscillations of the Atoms and Superfluidity

The main features of superfluidity of liquid helium became clear few decades ago [26], [27]. L. D. Landau explains this phenomenon as the manifestation of a quantum behavior of the macroscopic object.

However, the causes and mechanism of the formation of superfluidity are not clear till our days. There is no explanation why the λ-transition in helium-4 occurs at about 2 K, that is about twice less than its boiling point:

$$\frac{T_{boiling}}{T_\lambda} \approx 1.94, \tag{11.1}$$

while for helium-3, this transition is observed only at temperatures about a thousand times smaller.

The related phenomenon, superconductivity, can be regarded as superfluidity of a charged liquid. It can be quantitatively described considering it as the consequence of ordering of zero-point oscillations of electron gas. Therefore it seems appropriate to consider superfluidity from the same point of view [54].

Atoms in liquid helium-4 are electrically neutral, as they have no dipole moments and do not form molecules. Yet some electromagnetic mechanism should be responsible for phase transformations of liquid helium (as well as in other condensed substance where phase transformations are related to the changes of energy of the same scale).

F. London has demonstrated already in the 1930's [53], that there is an interaction between atoms in the ground state, and this interaction is of a quantum nature. It can be considered as a kind of the Van-der-Waals interaction. Atoms in their ground state (T = 0) perform zero-point oscillations. F.London was considering vibrating atoms as three-dimensional oscillating dipoles which are connected to each other by the electromagnetic interaction. He proposed to call this interaction as the dispersion interaction of atoms in the ground state.

11.2   The Dispersion Effect in Interaction of Atoms in the Ground State

Following F. London [53], let us consider two spherically symmetric atoms without non-zero average dipole moments. Let us suppose that at some time the charges of these atoms are fluctuationally displaced from the equilibrium states:

$$
\begin{cases}
r_1 = (x_1, y_1, z_1) \\
r_2 = (x_2, y_2, z_2)
\end{cases}
\tag{11.2}
$$

If atoms are located along the Z-axis at the distance L of each other, their potential energy can be written as:

$$
\mathcal{H} = \underbrace{\frac{e^2 r_1^2}{2a} + \frac{e^2 r_2^2}{2a}}_{elastic\ dipoles\ energy} + \underbrace{\frac{e^2}{L^3}(x_1 x_2 + y_1 y_2 - 2z_1 z_2)}_{elastic\ dipoles\ interaction}.
\tag{11.3}
$$

where a is the atom polarizability.

The Hamiltonian can be diagonalized by using the normal coordinates of symmetric and antisymmetric displacements:

$$r_s \equiv \begin{cases} x_s = \frac{1}{\sqrt{2}}(x_1 + x_2) \\ y_s = \frac{1}{\sqrt{2}}(y_1 + y_2) \\ z_s = \frac{1}{\sqrt{2}}(z_1 + z_2) \end{cases}$$

and

$$r_a \equiv \begin{cases} x_a = \frac{1}{\sqrt{2}}(x_1 - x_2) \\ y_a = \frac{1}{\sqrt{2}}(y_1 - y_2) \\ z_a = \frac{1}{\sqrt{2}}(z_1 - z_2) \end{cases}$$

This yields

$$x_1 = \frac{1}{\sqrt{2}}(x_s + x_a)$$
$$y_1 = \frac{1}{\sqrt{2}}(y_s + y_a)$$
$$z_1 = \frac{1}{\sqrt{2}}(z_s + z_a)$$

and

$$x_2 = \frac{1}{\sqrt{2}}(x_s - x_a)$$
$$y_2 = \frac{1}{\sqrt{2}}(y_s - y_a)$$
$$z_2 = \frac{1}{\sqrt{2}}(z_s - z_a)$$

As the result of this change of variables we obtain:

$$\begin{aligned}
\mathcal{H} &= \frac{e^2}{2a}(r_s^2 + r_a^2) + \frac{e^2}{2L^3}(x_s^2 + y_s^3 - 2z_s^2 - x_a^2 - y_a^2 + 2z_a^2) \\
&= \frac{e^2}{2a}\left[\left(1 + \frac{a}{L^3}\right)(x_s^2 + y_s^2) + \left(1 - \frac{a}{L^3}\right)(x_a^2 + y_a^2) \right. \\
&\quad \left. + \left(1 - 2\frac{a}{L^3}\right)z_s^2 + \left(1 + 2\frac{a}{L^3}\right)z_a^2 \right].
\end{aligned} \tag{11.4}$$

Consequently, frequencies of oscillators depend on their orientation and they are determined by the equations:

$$\overset{s}{\underset{a}{\Omega}}_{0x} = \overset{s}{\underset{a}{\Omega}}_{0y} = \Omega_0 \sqrt{1 \pm \frac{a}{L^3}} \approx \Omega_0 \left(1 \pm \frac{a}{L^3} - \frac{a^2}{8L^6} \pm ...\right),$$

$$\overset{s}{\underset{a}{\Omega}}_{0z} = \Omega_0 \sqrt{1 \mp \frac{2a}{L^3}} \approx \Omega_0 \left(1 \mp \frac{a}{L^3} - \frac{a^2}{2L^6} \mp ...\right), \tag{11.5}$$

where

$$\Omega_0 = \frac{2\pi e}{\sqrt{ma}} \tag{11.6}$$

is natural frequency of the electronic shell of the atom (at $L \to \infty$). The energy of zero-point oscillations is

$$\mathcal{E} = \frac{1}{2}\hbar(\Omega_0^s + \Omega_0^a) \tag{11.7}$$

It is easy to see that the description of interactions between neutral atoms do not contain terms $\frac{1}{L^3}$, which are characteristics for the interaction of zero-point oscillations in the electron gas (Eq.(6.7)) and which are responsible for the occurrence of superconductivity. The terms that are proportional to $\frac{1}{L^6}$ manifest themselves in interactions of neutral atoms.

It is important to emphasize that the energies of interaction are different for different orientations of zero-point oscillations. So the interaction of zero-point oscillations oriented along the direction connecting the atoms leads to their attraction with energy:

$$\mathcal{E}_z = -\frac{1}{2}\hbar\Omega_0 \frac{A^2}{L^6}, \tag{11.8}$$

while the sum energy of the attraction of the oscillators of the perpendicular directions (x and y) is equal to one half of it:

$$\mathcal{E}_{x+y} = -\frac{1}{4}\hbar\Omega_0 \frac{A^2}{L^6} \tag{11.9}$$

(the minus sign is taken here because for this case the opposite direction of dipoles is energetically favorable).

11.3 The Estimation of Main Characteristic Parameters of Superfluid Helium

11.3.1 The Main Characteristic Parameters of the Zero-Point Oscillations of Atoms in Superfluid Helium-4

There is no repulsion in a gas of neutral bosons. Therefore, due to attraction between the atoms at temperatures below

$$T_{boil} = \frac{2}{3k}\mathcal{E}_z \tag{11.10}$$

this gas collapses and a liquid forms.

At twice lower temperature

$$T_\lambda = \frac{2}{3k}\mathcal{E}_{x+y} \tag{11.11}$$

all zero-point oscillations become ordered. It creates an additional attraction and forms a single quantum ensemble.

A density of the boson condensate is limited by zero-point oscillations of its atoms. At condensation the distances between the atoms become approximately equal to amplitudes of zero-point oscillations.

Coming from it, we can calculate the basic properties of an ensemble of atoms with ordered zero-point oscillations, and compare them with measurement properties of superfluid helium.

We can assume that the radius of a helium atom is equal to the Bohr radius a_B, as it follows from quantum-mechanical calculations. Therefore, the energy of electrons on the s-shell of this atom can be considered to be equal:

$$\hbar\Omega_0 = \frac{4e^2}{a_B} \tag{11.12}$$

As the polarizability of atom is approximately equal to its volume [56]

$$A \simeq a_B^3, \tag{11.13}$$

the potential energy of dispersive interaction (11.9), which causes the ordering zero-point oscillations in the ensemble of atoms, we can represent by the equation:

$$\mathcal{E}_{x+y} = -\frac{e^2}{a_B}a_B^6 n^2, \tag{11.14}$$

where the density of helium atoms

$$n = \frac{1}{L^3} \tag{11.15}$$

The velocity of zero-point oscillations of helium atom. It is naturally to suppose that zero-point oscillations of atoms are harmonic and the equality of kinetic and potential energies are characteristic for them:

$$\frac{M_4\widehat{v_0}^2}{2} - \frac{e^2}{a_B}a_B^6 n^2 = 0, \tag{11.16}$$

where M_4 is mass of helium atom, $\widehat{v_0}$ is their averaged velocity of harmonic zero-point oscillations.

Hence, after simple transformations we obtain:

$$\widehat{v_0} = c\alpha^3 \left\{ \frac{n}{n_0} \right\}, \tag{11.17}$$

where the notation is introduced:

$$n_0 = \frac{\alpha^2}{a_B^3} \sqrt{\frac{M_4}{2m_e}}. \tag{11.18}$$

If the expression in the curly brackets

$$\frac{n}{n_0} = 1, \tag{11.19}$$

we obtain

$$\widehat{v_0} = c\alpha^3 \cong 116.5 \ m/s. \tag{11.20}$$

The density of liquid helium. The condition (11.19) can be considered as the definition of the density of helium atoms in the superfluid state:

$$n = n_0 = \frac{\alpha^2}{a_B^3} \sqrt{\frac{M_4}{2m_e}} \cong 2.172 \cdot 10^{22} \ atom/cm^3. \tag{11.21}$$

According to this definition, the density of liquid helium-4

$$\gamma_4 = nM_4 \cong 0.1443 \ g/cm^3 \tag{11.22}$$

that is in good agreement with the measured density of the liquid helium $0.145 \ g/cm^3$ for $T \simeq T_\lambda$.

Similar calculations for liquid helium-3 gives the density $0.094 \ g/cm^3$, which can be regarded as consistent with its density $0.082 \ g/cm^3$ experimentally measured near the boiling point.

The dielectric constant of liquid helium. To estimate the dielectric constant of helium we can use the Clausius-Mossotti equation [56]:

$$\frac{\varepsilon - 1}{\varepsilon + 2} = \frac{4\pi}{3} nA. \tag{11.23}$$

At taking into account Eq.(11.13), we obtain

$$\varepsilon \approx 1.040, \tag{11.24}$$

that differs slightly from the dielectric constant of the liquid helium, measured near the λ-point [57]:

$$\varepsilon \approx 1.057 \tag{11.25}$$

The temperature of λ-point. The superfluidity is destroyed at the temperature T_λ, at which the energy of thermal motion is compared with the energy of the Van-der-Waals bond in superfluid condensate

$$\frac{3}{2}kT_\lambda - \frac{e^2}{a_B}a_B^6 n^2 = 0. \tag{11.26}$$

With taking into account Eq.(11.21)

$$T_\lambda = \frac{1}{3k}\frac{M_4}{m_e}\frac{\alpha^4 e^2}{a_B} \tag{11.27}$$

or after appropriate substitutions

$$T_\lambda = \frac{1}{3}\frac{M_4 c^2 \alpha^6}{k} = 2.177K, \tag{11.28}$$

that is in very good agreement with the measured value $T_\lambda = 2.172K$.

The boiling temperature of liquid helium. After comparison of Eq.(11.8) - Eq.(11.9), we have

$$T_{boil} = 2T_\lambda = 4.35K \tag{11.29}$$

This is the basis for the assumption that the liquefaction of helium is due to the attractive forces between the atoms with ordered lengthwise components of their oscillations.

The velocity of the first sound in liquid helium. It is known from the theory of the harmonic oscillator that the maximum value of its velocity is twice bigger than its average velocity. In this connection, at assumption that the first sound speed c_{s1} is limited by this maximum speed oscillator, we obtain

$$c_{s1} = 2\widehat{v_0} \simeq 233 \ m/s. \tag{11.30}$$

It is in consistent with the measured value of the velocity of the first sound in helium, which has the maximum value of 238.3 m/s at $T \to 0$ and decreases with increasing temperature up to about 220 m/s at $T = T_\lambda$.

The results obtained in this section are summarized for clarity in the Table 11.1.

The measurement data in this table are mainly quoted by [55] and [57].

Table 11.1   *Comparison of the calculated values of liquid helium-4 with the measurement data.*

parameter	defining formula	calculated value	measured value
the velocity of zero-point oscillations of helium atom	$\widehat{v_0} = c\alpha^3$	116.5 m/s	
The density of atoms in liquid helium	$n = \sqrt{\frac{M_4}{2m_e}}\frac{\alpha^2}{a_B^3}$	$2.172 \cdot 10^{22}$ $atom/cm^3$	
The density of liquid helium-4 g/l	$\gamma = M_4 n$	144.3	$145_{T \simeq T_\lambda}$
The dielectric constant of liquid helium-4	$\frac{\varepsilon - 1}{\varepsilon + 2} = \frac{4\pi}{3}\alpha^2\sqrt{\frac{M_4}{2m_e}}$	1.040	$1.048_{T \simeq 4.2}$ $1.057_{T \simeq T_\lambda}$
The temperature λ-point,K	$T_\lambda \simeq \frac{M_4 c^2 \alpha^6}{3}$	2.177	2.172
The boiling temperature of helium-4,K	$T_{boil} \simeq 2T_\lambda$	4.35	4.21
The first sound velocity, m/s	$c_{s1} = 2\widehat{v_0}$	233	$238.3_{T \to 0}$

11.3.2   The Estimation of Characteristic Properties of He-3

The estimation of characteristic properties of He-3. The results of similar calculations for the helium-3 properties are summarized in the Table 11.2.

Table 11.2   *The characteristic properties of liquid helium-3.*

parameter	defining formula	calculated value	measured value
The velocity of zero-point oscillations of helium atom	$\widehat{v_0} = c\alpha^3$	116.5 m/s	
The density of atoms in liquid helium-3	$n_3 = \sqrt{\frac{M_3}{2m_e}}\frac{\alpha^2}{a_B^3}$	$1.88 \cdot 10^{22}$ $atom/cm^3$	
The density of liquid helium-3, g/l	$\gamma = M_3 n_3$	93.7	82.3
The dielectric constant of liquid helium-3	$\frac{\varepsilon-1}{\varepsilon+2} = \frac{4\pi}{3}\alpha^2\sqrt{\frac{M_3}{2m_e}}$	1.035	
The boiling temperature of helium-3,K	$T_{boil} \simeq \frac{4}{3}\frac{\mathcal{E}_{x+y}}{k}$	3.27	3.19
The sound velocity in liquid helium-3	$c_s = 2\widehat{v_0}$	233 m/s	

There is a radical difference between mechanisms of transition to the superfluid state for He-3 and He-4. Superfluidity occurs if complete ordering exists in the atomic system. For superfluidity of He-3 electromagnetic interaction should order not only zero-point vibrations of atoms, but also the magnetic moments of the nuclei.

It is important to note that all characteristic dimensions of this task: the amplitude of the zero-point oscillations, the atomic radius, the distance between atoms in liquid helium - all equal to the Bohr radius a_B by the order of magnitude. Due to this fact, we can estimate the oscillating magnetic field, which a fluctuating electronic shell creates on "its" nucleus:

$$H_\Omega \approx \frac{e}{a_B^2} \frac{a_B \Omega_0}{c} \approx \frac{\mu_B}{A_3}, \tag{11.31}$$

where $\mu_B = \frac{e\hbar}{2m_e c}$ is the Bohr magneton, A_3 is the electric polarizability of helium-3 atom.

Because the value of magnetic moments for the nuclei He-3 is approximately equal to the nuclear Bohr magneton $\mu_{n_B} = \frac{e\hbar}{2m_p c}$, the ordering in their system must occur below the critical temperature

$$T_c = \frac{\mu_{n_B} H_\Omega}{k} \approx 10^{-3} K. \tag{11.32}$$

This finding is in agreement with the measurement data. The fact that the nuclear moments can be arranged in parallel or antiparallel to each other is consistent with the presence of the respective phases of superfluid helium-3.

Concluding this approach permits to explain the mechanism of superfluidity in liquid helium.

In this way, the quantitative estimations of main parameters of the liquid helium and its transition to the superfluid state were obtained.

It was established that the phenomenon of superfluidity as well as the phenomenon of superconductivity is based on the physical mechanism of the ordering of zero-point oscillations.

Part IV

Conclusion

Chapter 12

Consequences

12.1 Super-Phenomena

Until now it has been commonly thought that the existence of the isotope effect in superconductors leaves only one way for explanation of the superconductivity phenomenon - the way based on the phonon mechanism.

Over fifty years of theory development based on the phonon mechanism, has not lead to success. All attempts to explain why some superconductors have certain critical temperatures (and critical magnetic fields) have failed.

This problem was further exacerbated with the discovery of high temperature superconductors. How can we move forward in HTSC understanding, if we cannot understand the mechanism that determines the critical temperature elementary superconductors?

In recent decades, experimenters have shown that isotopic substitution in metals leads to a change in the parameters of their crystal lattice and thereby affect the Fermi energy of the metal. As results, the superconductivity can be based on a nonphonon mechanism.

The theory proposed in this paper suggests that the specificity of the association

mechanism of electrons pairing is not essential. It is merely important that such a mechanism was operational over the whole considered range of temperatures. The nature of the mechanism forming the electron pairs does not matter, because although the work of this mechanism is necessary it is still not a sufficient condition for the superconducting condensate's existence. This is caused by the fact that after the electron pairing, they still remain as non-identical particles and cannot form the condensate, because the individual pairs differ from each other as they commit uncorrelated zero-point oscillations. Only after an ordering of these zero-point oscillations, an energetically favorable lowering of the energy can be reached and a condensate at the level of minimum energy can then be formed. Due to this reason the ordering of zero-point oscillations must be considered as the cause of the occurrence of superconductivity.

Therefore, the density of superconducting carriers and the critical temperature of a superconductor are determined by the Fermi energy of the metal, The critical magnetic field of a superconductor is given by the mechanism of destruction of the coherence of zero-point oscillations.

In conclusion, the consideration of zero-point oscillations allows us to construct the theory of superconductivity, which is characterized by the ability to give estimations for the critical parameters of elementary superconductors. These results are in satisfactory agreement with measured data.

This approach permit to explain the mechanism of superfluidity in liquid helium. For electron shells of atoms in S-states, the energy of interaction of zero-point oscillations can be considered as a manifestation of Van-der-Waals forces. In this way the apposite quantitative estimations of temperatures of the helium liquefaction and its transition to the superfluid state was obtained.

Thus it is established that both related phenomena, superconductivity and superfluidity, are based on the same physical mechanism - they both are consequences of the ordering of zero-point oscillations.

12.2   A Little More About Pseudo-Theories

It is important not only that the number of pseudo-theories emerged in the twentieth century, but more important is the fact that they are long-lived and continue to exist today. It would seem to be all clear to them since they do not satisfy the main principle of the natural sciences. One might think that the editorial boards of scientific physics journals can be blamed in it. The most of reviewers in these journals are theorists naturally. Often they developed their own criterion of the correctness of a particular article. They believe in their own theories, and not allowed to publish articles that are not consistent with these theories, even if it is obvious that the models in these studies are consistent with the measured data.

As pseudo-theories violate the Gilbert's-Galileo's basic principle, apparently, the editors of these journals need to mark this moment. It probably makes sense to open in professional journals special columns with the name of the type "Hypothetical studies which at this stage has not yet satisfy the general principle of the natural science" and to publish on pages of these columns results of studies that in according with our systematization must be placed in the cell 4 of Table 1.1. In this case, the readers and the Nobel committee will be easier to develop a cautious attitude to these theories.

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