Chapter 7

On Certain Class of Zweier I-Convergent Sequence Spaces

7.1 Introduction

Theorem 7.1.1. [68, Theorem 2.1] The sets $\mathcal{Z}$ and $\mathcal{Z}_0$ are the linear spaces with the co-ordinate wise addition and scalar multiplication which are the BK-spaces with the norm

$$||x||_{\mathcal{Z}} = ||x||_{\mathcal{Z}_0} = ||Z^p x||_c.$$

Theorem 7.1.2. [68, Theorem 2.2] The sequence spaces $\mathcal{Z}$ and $\mathcal{Z}_0$ are linearly isomorphic to the spaces c and c_0 respectively, i.e $\mathcal{Z} \cong c$ and $\mathcal{Z}_0 \cong c_0$.

Theorem 1.3. [68, Theorem 2.3] The inclusions $\mathcal{Z}_0 \subset \mathcal{Z}$ strictly hold for $p \neq 1$.

7.2 Main Results

Recently Šalát, Tripathy and Ziman[65-66] introduced the following sequence spaces

$$c_0^I = \{(x_k) \in \omega : \{k \in \mathbb{N} : |x_k| \geq \epsilon\} \in I\},$$

$$c^I = \{(x_k) \in \omega : \{k \in \mathbb{N} : |x_k - L| \geq \epsilon\} \in I, \text{for some L} \in \mathbb{C} \},$$

$$\ell_\infty^I = \{(x_k) \in \omega : \{k \in \mathbb{N} : |x_k| \geq M\} \in I, \text{ for each fixed M>0}\}.$$

Analogous to Kostyrko, Šalát and Wilczyński[12], Šalát Tripathy and Ziman[65-66], Khan and Ebadullah[29,31,37,38] introduced the following classes of sequences.

$$\mathcal{Z}_0^I = \{x = (x_k) \in \omega : \{k \in \mathbb{N} : I - \lim Z^p x = 0\} \in I\},$$

"If I feel unhappy, I do mathematics to become happy. If I feel happy, I do mathematics to keep happy." -Paul Turan

$$\mathcal{Z}^I = \{x = (x_k) \in \omega : \{k \in \mathbb{N} : I - \lim Z^p x = L, \text{ for some L}\} \in I\},$$

$$\mathcal{Z}^I_\infty = \{x = (x_k) \in \omega : \sup_k |Z^p x| < \infty\}.$$

In [27] for $q = (q_k)$ a sequence of positive reals

$$\mathcal{Z}^I_0(q) = \{x = (x_k) \in \omega : \{k \in \mathbb{N} : |Z^p x|^{q_k} \geq \epsilon\} \in I\},$$

$$\mathcal{Z}^I(q) = \{x = (x_k) \in \omega : \{k \in \mathbb{N} : |Z^p x - L|^{q_k} \geq \epsilon\} \in I, \text{ for some L} \in \mathbb{C}\ \},$$

$$\mathcal{Z}^I_\infty(q) = \{x = (x_k) \in \omega : \sup_k |Z^p x|^{q_k} < \infty\}.$$

In [8] for an Orlicz function M and $Z^p x = x^/$

$$\mathcal{Z}^I_0(M) = \{x = (x_k) \in \omega : I - \lim M(\frac{|x^/_k|}{\rho}) = 0 \text{ for some } \rho > 0\},$$

$$\mathcal{Z}^I(M) = \{x = (x_k) \in \omega : I{-}\lim M(\frac{|x^/_k - L|}{\rho}) = 0 \text{ for some L and } \rho > 0\},$$

$$\mathcal{Z}^I_\infty(M) = \{x = (x_k) \in \omega : \sup_k M(\frac{|x^/_k|}{\rho}) < \infty \text{ for some } \rho > 0\}.$$

In [29] for a modulus function f

$$\mathcal{Z}^I_0(f) = \{(x_k) \in \omega : \text{for a given } \varepsilon > 0, \{k \in \mathbb{N} : f(|x^/_k|) \geq \varepsilon\} \in I\},$$

$$\mathcal{Z}^I(f) = \{(x_k) \in \omega : \text{there is } L \in \mathbb{C} \text{ such that}$$
$$\text{for } \varepsilon > 0, \{k \in \mathbb{N} : f(|x^/_k - L|) \geq \varepsilon\} \in I\},$$

$$\mathcal{Z}^I_\infty(f) = \{(x_k) \in \omega : \{k \in \mathbb{N} : f(|x^/_k|) \geq M\} \in I, \text{ for each fixed M>0}\}.$$

In [34] for a sequence of modulii $F = (f_k)$

$$\mathcal{Z}^I_0(F) = \{(x_k) \in \omega : \{k \in \mathbb{N} : f_k(|x^/_k|) \geq \varepsilon\} \in I\},$$

$$\mathcal{Z}^I(F) = \{(x_k) \in \omega : \{k \in \mathbb{N} : f_k(|x^/_k - L|) \geq \varepsilon, \text{ for some L} \in \mathbb{C}\ \} \in I\},$$

$$\mathcal{Z}_{\infty}^{I}(F) = \{(x_k) \in \omega : \{k \in \mathbb{N} : f_k(|x_k^{/}|) \geq M, \text{for each fixed M>0}\} \in I\}.$$

Here we give the canonical inclusion relations

Result 7.2.1. $c_0^I \subset c^I \subset \ell_{\infty}^I$.(See[41,57,58]).

Result 7.2.2. $\mathcal{Z}_0^I \subset \mathcal{Z}^I \subset \mathcal{Z}_{\infty}^I$.(See[12]).

Result 7.2.3. $\mathcal{Z}_0^I(q) \subset \mathcal{Z}^I(q) \subset \mathcal{Z}_{\infty}^I(q)$.(See[27]).

Result 7.2.4. $\mathcal{Z}_0^I(M) \subset \mathcal{Z}^I(M) \subset \mathcal{Z}_{\infty}^I(M)$.(See[35]).

Result 7.2.5. $\mathcal{Z}_0^I(f) \subset \mathcal{Z}^I(f) \subset \mathcal{Z}_{\infty}^I(f)$.(See[29]).

Result 7.2.6. $\mathcal{Z}_0^I(F) \subset \mathcal{Z}^I(F) \subset \mathcal{Z}_{\infty}^I(F)$.