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Zweier I - Convergent Sequence Spaces and Their Properties

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Preface

Sequence spaces play an important role in various fields of Real Analysis, Complex Analysis, Functional Analysis and Topology. These are very useful tools in demonstrating abstract concepts through constructing examples and counter examples. The topic “Sequence Spaces” is very broad in its own sense as one can study from various point of views, e.g. Schauder decomposition, α -, β -, and γ - duals, matrix transformations, measures of noncompactness, topological properties and geometric properties. The central theme of the present book is to introduce and study Zweier I-Convergent sequence spaces.

The structure of this text is straightforward. There are six chapters devoted to the various aspects of the theory. Each chapter is divided into sections. The numbers in the square brackets refers to the references listed in the bibliography.

As usual chapter 1, is devoted to the background materials which begins with the notations and conventions and some basic definitions which are needed throughout the work. This chapter concludes with an introduction to the Ideals which also includes some elementary properties.

In chapter 2, we introduce the Zweier I-convergent sequence spaces \mathcal{Z}^I , \mathcal{Z}_0^I and \mathcal{Z}_∞^I . We prove the decomposition theorem and study topological, algebraic properties and inclusion relations of these spaces.

In chapter 3, we introduce the Paranorm Zweier I-convergent sequence spaces $\mathcal{Z}^I(q)$, $\mathcal{Z}_0^I(q)$ and $\mathcal{Z}_\infty^I(q)$ for $q = (q_k)$, a sequence of positive real numbers. We study some topological properties, prove the decomposition theorem and study some inclusion relations on these spaces.

In chapter 4, we introduce the sequence spaces $\mathcal{Z}^I(M)$, $\mathcal{Z}_0^I(M)$ and $\mathcal{Z}_\infty^I(M)$ using the Orlicz function M . We study the algebraic properties and inclusion relations on these spaces.

In chapter 5, we introduce the sequence spaces $\mathcal{Z}^I(f)$, $\mathcal{Z}_0^I(f)$ and $\mathcal{Z}_\infty^I(f)$ for a modulus function f and study some of the topological and algebraic properties on these spaces.

Preface

In chapter 6, we introduce the sequence spaces $\mathcal{Z}^I(F)$, $\mathcal{Z}_0^I(F)$ and $\mathcal{Z}_\infty^I(F)$ for a sequence of moduli $F = (f_k)$ and study some of the topological and algebraic properties on these spaces.

In chapter 7, This is a precise chapter which is very special as it is designed only to study some inclusion relations between various zweier sequence spaces studied previously.

In chapter 8, we introduce the sequence spaces ${}_2\mathcal{Z}^I(F)$, ${}_2\mathcal{Z}_0^I(F)$ and ${}_2\mathcal{Z}_\infty^I(F)$ for a sequence of moduli $F = (f_k)$ and study some of the topological and algebraic properties on these spaces.

In chapter 9, we introduce the sequence spaces ${}_2\mathcal{Z}^I(f)$, ${}_2\mathcal{Z}_0^I(f)$ and ${}_2\mathcal{Z}_\infty^I(f)$ for a modulus function f and study some of the topological and algebraic properties on these spaces.

In chapter 10, we introduce the sequence spaces ${}_2\mathcal{Z}^I(M)$, ${}_2\mathcal{Z}_0^I(M)$, ${}_2\mathcal{Z}_\infty^I(M)$ for an Orlicz function M and study some of the topological and algebraic properties on these spaces.

The book ends with a fairly exhaustive bibliography of books and research articles consulted for the work.

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Contents

1	Basic Definitions and Notations	1
2	Zweier I-Convergent Sequence Spaces	19
2.1	Introduction	21
2.2	Main Results	23
3	On Paranorm Zweier I-Convergent Sequence Spaces	33
3.1	Introduction	35
3.2	Main Results	36
4	Zweier I-Convergent Sequence Spaces Defined by Orlicz Function	47
4.1	Introduction	49
4.2	Main Results	50
5	On Some Zweier I-Convergent Sequence Spaces Defined by a Modulus Function	57
5.1	Introduction	59

Contents

5.2	Main Results	60
6	Zweier I-Convergent Sequence Spaces Defined by a Sequence of Moduli	69
6.1	Introduction	71
6.2	Main Results	72
7	On Certain Class of Zweier I-Convergent Sequence Spaces	81
7.1	Introduction	83
7.2	Main Results	83
8	Zweier I-Convergent Double Sequence Spaces	87
8.1	Introduction	89
8.2	Main Results	91
9	Zweier I-Convergent Double Sequence Spaces Defined by a Modulus Function	101
9.1	Introduction	103
9.2	Main Results	104
10	Zweier I-Convergent Double Sequence Spaces Defined by Orlicz Function	113
10.1	Introduction	115
10.2	Main Results	115