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The Basis of Unitary Quantum Theory (UQT)

1.1 Some Difficulties in Standard Quantum Theory and Ideas for a New Approach

It is difficult, if not impossible; to avoid the conclusion that only mathematical description expresses all our knowledge about the various aspects of our reality.

- an opinion extracted from a Soviet newspaper

Over seventy-five years have passed since the field of quantum mechanics emerged. Each day, the experiments being done with huge particle accelerators reveal new details about the design of microcosmic structures, and supercomputers crunch vast quantities of resulting mathematical data. But till now we do not have any theoretical approach to the determination of the mass spectrum of elementary particles with number more than 750, more over we do not yet fully understand the strong interaction itself. The standard quantum theory avoids the physical descriptions of various phenomena in terms of images and movements. Many different approaches have been taken to develop a quantum field theory, but typically the divergences have created provoke abundant nightmares for theoretical physicists. Nevertheless, we'll try to classify and formalize these approaches somewhat below.

Let us begin with the common canonical point of view based on the properties of space-time, particles, and the vacuum, on particle interactions, and on mathematical modelling equations. Every postulate of canonical theory may be reduced to the following seven statements (not all of which are without issues):

1. Space-time is four-dimensional, continuous, homogeneous, and isotropic.

2. Particles and their interactions are local.

3. There is only one vacuum and it is non-degenerating.

4. It is a valid proposition in quantum theory that physical values correspond to Hermitian operators and that the physical state corresponds to vectors in Gilbert space with positively determined metrics.

5. The requirement of relativistic invariance is imposed (four-dimensional rotation with coordinate translation – Poincaré group).

6. The equations for non-interacting free particles are linear and do not contain derivatives higher than of the second order.

7. Particles' internal characteristics of symmetry are described with the SU2 and SU3 symmetry groups.

The previous statements provide the basis for the construction of the S-matrix, that describes the transformation of one asymptotic state into another and satisfies the conditions of causality and unity. Nevertheless, this approach, which seems mathematically excellent in outward appearance, still leads to divergences. Recent 'normalized' theories, derived to provide a means of avoiding infinities by one technique or another, sometimes end up seeming more like circus tricks.

We shall not criticize such normalized theories here; however, to quote P. A. M. Dirac*:

"…most physicists are completely satisfied with the existing situation. They consider relativistic quantum field theory and electrodynamics to be quite perfect theories and it is not necessary to be anxious about the situation. I should say that I do not like that at all, because according to such 'perfect' theory we have to neglect, without any reason, infinities that appear

*in the equations. It is just mathematical nonsense. Usually in mathematics the value can be rejected only in the case it were too small, but not because it is infinitely big and someone would like to get rid of it.” * Direction in Physics, New York, 1978*

One can try to solve this problem by looking at it from the other side and forming a theory in such a way that it would not contain divergences at all. However, that way leads to the necessity to reject one or another thesis of the canonical point of view. In canonical theory, the appearance of divergences is caused by integrals connected with some of the particle parameters and considered in the whole of space, from zero to infinity, and for particles as points. The infinities appear by integration only in the region near zero, i.e., on an infinitesimal scale.

The elimination of divergences might be achieved within the purview of one or more of the following four different parameters or approaches in quantum theory:

1. the minimal elementary length is introduced and then the integration is carried out not from zero, and therefore all such integrals become finite;

2. it is considered that space-time is discontinuous, consisting entirely of separate points, whereby such a space-time model corresponds to a crystalline lattice. To get a discontinuous coordinate and time spectrum, time and coordinate operators are introduced (per quantized space-time theory);

3. non-linear equations containing derivatives of high order may be used instead of linear equations with derivatives of the first and second order only. Even more desperate measures are sometimes used: introduction of coordinate systems with indefinite metrics instead of coordinate systems with definite metrics;

4. it could be assumed that a particle is not a point, and hence a whole series of

non-local theories might be derived.

These four approaches have so far not yielded notable results, so another two techniques have been subsequently considered: enlargement of the Poincaré group, and generalization of internal symmetry groups.

Let us first discuss the problems connected with the enlargement of the Poincaré group, assuming in accordance with observations of natural phenomena that symmetries of sufficiently high level are realized. There are two such enlargement methods:

1. The Poincaré group is enlarged up to the conformal group, which includes scale and special conformal transformation in addition to the usual four-dimensional rotation (Lorentz group) and coordinate translations. However, if enlargement of the Poincaré group up to the conformal group is performed, then generators of the same tensor character should be added to the tensor generators of the Poincaré group's $M_{\mu\nu}$ (rotation) and P_{μ} (shifts). Unfortunately, after such enlargement the group multiplets contain either bosons or fermions only; in essence, these multiplets are not mixed. The worst situation is with the basic equation for particles. One can write such a conformal invariant equation only for particles with mass equal to zero. This situation may be improved with a new definition of mass (i.e., the so-called conformal mass is introduced), but thereafter its physical sense of particles becomes positively vague. To get out of a difficult situation in this case, attempts have been made to reject exact conformal invariance; then the mass appears as a result of conformal asymmetry violation. We have the same situation in the case of the SU3 symmetry group. This method was not successful.

2. Generators of the spinor type may be added to the enlarged Poincaré group. Such widening results in a new type of symmetry called ‘super-symmetry’. For that purpose, so-called super-space is introduced: an eight-dimensional space where the points are denoted as the common coordinates $x_\mu (\mu = 0, 1, 2, 3)$ of space-time and also the anti-commutating spinor θ with four components. In this case, the super-symmetry group may be considered as a transformation group of the newly introduced super-space. The super-symmetry group then includes special super-transformation in addition to four-dimensional rotation and coordinate translations (Poincaré group). Representations (multiplets) Ψ of the super-symmetry group depend both on x_μ and $\theta : \Psi(\theta)$ operators. These functions are called super-fields and contain both boson and fermion fields. In other words, super-symmetries, bosons, and fermion fields are mixed. However, within such super-multiplets all particles have equal masses. In addition, this model is far from ‘reality’, as the physical meaning of super-symmetry is absolutely vague.

Let us now examine the so called second approach to eliminating divergences, connected with the generalization of the internal symmetry group. The simplest and most widely used groups of internal symmetry are SU2 and SU3. Two such generalizations have been actively investigated: the chiral group and a group of local calibrating transformations.

1. The chiral groups are direct products of SU2 and SU3, yielding SU2 x SU2 and SU3 x SU3 groups. For the construction of a chiral symmetric Lagrangian are used either chiral group multiplets in the form of polynomial functions of the field operators and their derivatives (i.e., linear realization of

chiral symmetry), or the Lagrangian is constructed with a small number of fields in the form of non-polynomial functions (for nonlinear realization of the chiral symmetry). In this case, some interesting results have been obtained, but the divergence problem seems to remains 'infinitely' far from solution.

2. With regards to local calibrating transformations, usually standard calibrating transformations do not depend on the coordinates of space-time; in other words, they are global. If we now assume that calibrating transformations are different in different points of the space-time coordinate system, then they may be combined into the local calibrating transformations group. If the Lagrangian is invariant in relation to global calibrating transformations, it is non-invariant in relation to the local calibrating group. Now it is necessary to somehow compensate incipient non-invariance of the global Lagrangian to derive the local invariant Lagrangian from the global invariant. This is done by the introduction of special Yang-Mills fields or compensating fields.

However, only particles with zero-mass vector like photons correspond to the Yang-Mills fields. Lack of mass results simply from the calibrating transformation. To obtain particles with non-zero mass, the special mechanism of spontaneous symmetry breaking has been proposed. This mechanism is such that, although the Lagrangian remains calibrating-invariant, the overall vacuum average of some fields that are part of the Lagrangian differs from zero, and the vacuum becomes degenerate. But it is impossible to create a substance field by means of Yang-Mills fields, and the former must be separately introduced.

There are several variations in theoretical development of this idea, the most successful being the Glashow-Weinberg-Salam model. According to this model, particles acquire finite mass if the terms responsible for spontaneous symmetry breakdown are added to the Lagrangian, usually by a certain combination of scalar fields (i.e., Higgs mechanism). Unfortunately, even that method has an essential defect, in that divergences still occur. A way was found to eliminate these divergences, but the neutral fields disappeared as well. Nevertheless, that method is considered as the one most propitious, and therefore the special mathematical apparatus based on equations of group renormalization is intensively developed.

Fifty years ago, J. Shwinger calculated the exact value of the anomalous magnetic moment of the electron. It was the remarkable result of modern quantum field theory magnificently confirmed by experimental data. However, in our opinion, his theory did not yield further essential physical correlations. While many mathematicians may deal primarily with quantum field theory, they seem to be still far from a deep physical understanding of the problem.

As a ‘safe’ example to illustrate this situation, we are going to examine the non-linear theory of A. Eddington, M. Born, and L. Infeld, which was favourably received and was incorporated into many quantum theory courses. Normally the authority of these scientists is presumed absolute; however…

The well-known Maxwell-Lorentz equations which describe the location and movement of an electron in a corresponding electro-magnetic field are as follows:

$$rot\,\mathbf{H} - \frac{1}{c}\frac{\partial \mathbf{E}}{\partial t} = 4\pi\rho\frac{\mathbf{v}}{c}, \text{ where } \; div\,\mathbf{E} = 4\pi\rho,$$

$$rot\,\mathbf{E} + \frac{1}{c}\frac{\partial \mathbf{H}}{\partial t} = 0, \text{ where } \; div\,\mathbf{H} = 0.$$

If we consider the electromagnetic field as a ‘substance’ but not the continuum of charged particles that make up different bodies, and use electrodynamics as a basis for mechanics, then charged particles should be regarded as nodal points of the electromagnetic field. Their location and movement should be governed by the laws of electromagnetic field variations in space and time. Then the only thing that prevents us to represent electrons as non-extended particles is the fact that the connected field created by electrons, according to the old concept (or creating them, in accordance with the new one), becomes infinite at their corresponding nodal points. Consequently, their mass estimated by their electromagnetic energy or momentum becomes infinite also. Thus, to combine the dynamic electromagnetic field theory (as a mechanical properties carrier) with the notion of the electron being non-extended, we should modify the above-mentioned Maxwell-Lorentz equation in such a way that, in spite of charge concentration at nodal points, the electromagnetic field would be finite at an arbitrarily small distance from those points. At median distances from the centre of the particle the field should appear 'normal', corresponding to the experimental data. Such a theoretical modification was made in 1922 by A. Eddington and in 1933 by M. Born and L. Infeld.

For this purpose, charge and current densities in the first two Maxwell-Lorentz equations were considered equal to zero over all of space except "special" points intended to be the electron locations. Furthermore, the vectors E and H in the same equations were correspondingly changed:

$$\mathbf{D} = \varepsilon \mathbf{E};\ \ \mathbf{B} = \frac{1}{\mu}\mathbf{H}$$

where

$$\varepsilon = \frac{1}{\mu} = \frac{1}{\sqrt{1 - \frac{E^2 - H^2}{E_0^2}}}$$

Here, $E = \frac{e}{r_0^2}$ represents the maximum possible value of the electric field in the centre of the electron and parameter r0 is considered as the electron's effective radius. The solution of such equations gives the finite electron mass, calculated as total energy of the electric field created by the particle:

$$E = \frac{e}{\sqrt{r_0^4 + r^4}}$$

Actually, the electric field at $r >> r_0$ now behaves in a normal way. However, everything in such a theory, from beginning to end, is fundamentally wrong: In the spherically symmetric case (the only type of event under consideration), the electrostatic intensity ought to be zero in the center of the particle because E is a vector! One can find similar absurdities in numerous modern quantum field theory descriptions, but their authors are still with us.

As for us, we should learn from history, perhaps by considering two rather droll academic episodes connected with distinguished physicist Wolfgang Pauli (they are not generally mentioned in classic scientific literature). It is well known that Louis de Broglie heard crushing criticism from Pauli upon first report of his ideas –but he later received the Nobel Prize for them. (For some time after that incident, de Broglie didn't attend international conferences.) A bit later, Pauli rose in sharp opposition to the publication of the article by G. E. Uhlenbek and S. Goudsmit representing the basic concept of 'spin'. However, this did not prevent him from developing the same idea and obtaining similar fundamental results, for which he thereafter received the Nobel Prize!

In any case, the mathematical descriptions and exact predictions of numerous very different quantum effects were so impressive that physicists became proud of their quantum science to a point bordering on self-satisfaction and superciliousness. They stopped thinking about physical description of the underlying phenomena and concentrated on the mathematical descriptions only. However, many problems in quantum theory are still far from resolution.

The ideas developed in this book differ completely from the canonical approach and its previously described versions. Our own approach is non-local, wherein basic theses of standard quantum theory are modified accordingly, and until now no one seems to have investigated such a rearrangement of ideas.

To reiterate key basic premises of our Unitary Quantum Theory (from the Introduction):

According to standard quantum theory, any microparticle is described by a wave function with a probabilistic interpretation that cannot be obtained from the mathematical formalism of non-relativistic quantum theory but is postulated instead.

The particle is considered as a point, which is "the source of the field, bun cannot be reduced to the field". Nothing can really be said about that micro-particle's actual "structure".

According to UQT, such a particle is considered as a bunched field (cluster) or 32-component wave packet of partial waves with linear dispersion [1-9]. Dispersion can be chosen in such a way that the wave packet would be alternately disappear and reappear in movement. The envelopment of this process coincides with the quantum mechanical wave function. Such concept helps to construct the relativistic – invariant model of UQT. Due to that theory the particle/wave packet, regarded as a function of 4-velocity, is described by partial differential equation in matrix form with 32x32 matrix or by equivalent partial differential system of 32 order. The probabilistic approach to wave function is not postulated, like it was earlier, but strictly results from mathematical formalism of the theory.

Particle mass is replaced in the UQT equation system with the integral over the whole volume of the bilinear field combinations, yielding a system of 32 integral-differential equations. In the scalar case the authors were able to calculate with 0.3% accuracy the non-dimensional electric charge and the constant of thin structure (see 1.4).

Electric charge quantization emerges as the result of a balance between dispersion and nonlinearity. Since the influence of dispersion is opposite to that of nonlinearity, for certain wave packet types the mutual compensation of these processes is possible.

The moving wave packet periodically appears and disappears at the de Broglie wavelength, but retains its form. A similar phenomenon may correspond to the theoretical case of oscillating solutions, as yet non-investigated mathematically.

Micro-particle birth and disintegration mechanisms become readily understood as the reintegrating and splitting-up of partial wave packets. This approach regards all interactions and processes as being simply a result of the mutual diffraction and interference of such wave packets, due to nonlinearity.

The tunnelling effect completely loses within UQT its mysteriousness. When the particle approaches the potential barrier in such the phase where the amplitude of the wave packet is small, all the equations become linear and the particle does not even “notice” the barrier, and if the phase corresponds to large packet’s amplitude, then nonlinear interaction begins and the particle can be reflected.

The most important result of our new Unitary Quantum Theory approach is the emergence of a general field basis for the whole of physical science, since the operational description of physical phenomena inherent in standard relativistic quantum theory is so wholly unsatisfying.

1.2 Further Inadequacies of Standard Quantum Mechanics and the Essence of a New Paradigm

Ernst Mach's outlook is well characterized by an episode from his life. Mach was studying ballistics and was often presented on the shooting grounds. Once he said to a colleague: "There is a question, which is constantly torturing me: Does the shell exist in the interval between the shooting and the hitting of the target? We do not see or feel it in any way."

"You are crazy," his colleague answered; "How can you doubt the existence of the shell? You yourself are calculating its trajectory, and your calculations agree with the experiment. Is this not proof of the shell's existence?"

"It does not prove anything," Mach objected. "The trajectory might only be a supplementary mathematical notion serving to predict further observations. The shell might not be moving along the trajectory at all. It might disappear at the moment of the shooting and reappear again at the moment it hits the target."

The colleague only shrugged his shoulders in surprise. But Mach did not stop there. In order to solve this problem he designed a special device for photographing the shell in flight. Mach was not only convinced that the shell existed in flight, but he also saw on the photos certain lines coming from the shell, which were called Mach lines.

It was due to his doubts about the existence of an unobserved flying shell that Mach created the supersonic gas dynamic theory. As a tribute to his achievements, the ratio of a flying *object's speed to the speed of sound is called the Mach number.*

H. Laitko and D. Hoffman,
Matters of Natural and Technical History, 1988 (4th), pp. 45-57.

Authors' note: The previous story happened long before quantum mechanics as well Newton comes to mind here with his theory of a 'quantum' ideology. It seems that new ideas often occur to the best researchers not in connection with any experimental data, and both stories would seem to confirm this well.

The most direct way of eliminating the existing theoretical difficulties in the relativistic interpretation of quantum-mechanical systems lies in the construction of a theory dealing only with a unified field, where the quantities and the values that characterize that field at different points in time and space are observed.

There is an impression that during the time since quantum theory was created, no substantial progress has been made in respect to our understanding of that theory. This impression is reinforced by the fact that neither field quantum theory nor the still imperfect theory of elementary particles made any serious strides in the posing or solution of the following traditional questions [1-3]:

- What are the reasons for the probabilistic interpretation of the wave function, and how can this interpretation be obtained from the mathematical formalism of the theory?

- What is really happening to a particle, when we "observe" it during interference experiments [for interference that cannot be explained without invoking the particle "splitting-up" concept]?

- What does this statement in standard quantum mechanics really mean?: *"a* *microparticle described by a point is the source of a field, but cannot be reduced to the field itself". Is it divisible or not? What does it really represent? Why everything in physics is based on two key notions: point-particle as the field source and the field itself? Can only one field aspect remains, and will physical entity be concidering as yet un-analyzable?*

There are as yet no answers to these basic questions. "Exorcism" of the complementarity principle is irrelevant because that philosophy was invented ad hoc.

Many researchers think that the future of theoretical physics should be based upon a certain single field theory – a unitary approach. In such a theory, particles are represented in the form of field wave clusters or packets. Mass would be purely a field notion, but the movement equations and all 'physical' inter-actions should follow directly from the field equations.

This book deals in more detail than Refs. [1-9, 165, 166, 170, 200, 201] with a very simple and heretofore unstudied possibility of formulating the unitary quantum theory for a single particle. Here we will deal only with the very general properties inherent in all particles and not touch upon the problems connected with such properties as charge, spin, strangeness, and charm.

After appearance and development of quantum mechanics, a curious situation occurred: half of the founders of the theory clearly spoke out against it! Their few remarks are given below:

"The existing quantum picture of material reality is today feebler and more doubtful than it has ever been. We know many interesting details and learn new ones every day. But we are still unable to select from the basic ideas one that could be regarded as certain and used as the foundation for a

stable construction. The popular opinion among the scientists proceeds from the fact that the objective picture of reality is impossible in its primary sense [i.e. in terms of images and movements- remark of authors]. Only very big optimists, among whom I count myself, take it is as philosophic exaltation, as a desperate step in the face of a large crisis. A solution of this crisis will ultimately lead to something better than the existing disorderly set of formulas forming the subject of quantum physics…If we are going to keep the damned quantum jumps I regret that I have dealt with quantum theory at all…" – Erwin Schroedinger

"The relativistic quantum theory as the foundation of modern science is fit for nothing." – P. A. M. Dirac

"Quantum physics urgently needs new images and ideas, which can appear only in case of a thorough review of its underlying principles." – Louis de Broglie

Albert Einstein, also, had the following to say:

"Great initial success of the quantum theory could not make me believe in a dice game being the basis of it…I do not believe this principal conception being an appropriate foundation for physics as a whole… Physicists think me an old fool, but I am convinced that the future development of physics will go in another direction than heretofore…I reject the main idea of modern statistical quantum theory… I'm quite sure that the existing statistical character of modern quantum theory should be ascribed to the fact that that theory operates with incomplete descriptions of physical systems only…"– A. Einstein

Although today the quantum theory is believed to be essentially correct in describing the phenomena of the micro-world, there is nevertheless experimental evidence of cold nuclear fusion and mass nuclear transmutations, of anomalous energy sources and perhaps even perpetual mobile—which contradicts quantum theory.

The ‘official’ quantum science does not believe in cold nuclear fusion phenomenon and regards people working within this sphere almost as charlatans. A good illustration of this is an article that appeared in Scientific American describing the annual awarding of jester Nobel prizes for completely phony works that generated a lot of furore. The article stated that the first candidates for the next jester Nobel Prize should be M. Fleishman and S. Pons (the discoverers of the cold nuclear fusion phenomenon). More than ten years have passed since then, but that prize has yet to be awarded!

We think that such an attitude of official science in the world is extreme and hostile toward all things new. The history of science abounds in remarkable examples of blindness and short-sightedness on the part of the official scientific establishment.

Here are some examples: When D. E. Mendeleev presented his periodic table to the Presidium of the Russian Academy of Sciences, the Vice-President (evidently, Academician Parrot) asked him: *“Mr. Mendeleev, did you try placing the elements in alphabetical order?”* We know that Mendeleev never became an academician!

N. I. Lobachevsky was dismissed from the position of Rector of the Kazan University as a “madman” for the construction of non-Euclidian geometry, and it would be naive to think that such things happen only in Russia. Let’s not forget the fires and inquisitions that took place in the scientific community more than 150 years ago.

Ernest Rutherford, the father of nuclear physics, thought that nuclear fission would be an exotic phenomenon largely unknown to the public. Heinrich Hertz, discoverer of electromagnetic waves, criticized researchers all over the world who were trying to use his discoveries for transmitting information, because he thought it to have no prospects whatsoever.

Such examples of blindness among the scientific community and its best representatives could be continued, but the foregoing should suffice. The history of the science shows clearly the validity of the objective dialectic law of the New struggling against Old. Intolerance and rigidity in the science, which the epigraph to this chapter well illustrates, can hardly do any good.

E. Mach was not the first to contemplate motion intermittence. Epicurus in his letter to Herodotus [10] wrote: *“The opinion that time intervals perceived in mind only contain continuous motion is wrong.”* Later that point of view was further developed and generalized by both Hindu and Arabic scholars. For example, according to Sautrantika teaching or doctrine [11] things appear from nothing, exist for a time, and then disappear again (!). In the same vein, Mutakallims asserted that everything in the world, all objects, properties, and even thought, change not continuously but in discrete steps: Things suddenly appear, exist within some time interval, and then similarly disappear to revive at another time and in another place – perhaps in a new form.

This principle, known in philosophy as that of ‘renovation’, was rediscovered by Leibnitz. In 1669, in a letter to his teacher and friend Thomasio, Leibnitz stated [12]: *“I have proven all that is moving is ceaselessly recreated, and that every body at any moment of motion is ‘something’; and at any time between these moments of motion it is nothing – an object unknown but essential.”*

The famous English mathematician W. Clifford also adhered to such a point of view. Philosopher Reichenbach wrote about similar phenomena [13] which should be described with adequate mathematical means, but it seems that no one has yet managed to accomplish it.

When the real phenomenon of corpuscular-wave dualism was discovered, the first idea that occurred to Schroedinger was to present the particle as a packet of de Broglie waves. Later, British mathematician C. G. Darwin [14] proved this idea to be wrong; as such wave packets would dissipate due to dispersion. Nevertheless, de Broglie studied the similar idea in a non-linear version, called the "double solution" (pilot wave) theory, until the end of his life.

The trouble with all previous attempts to present a particle as a field wave packet was that such a packet, according to proposed approaches, consisted of de Broglie waves. In our UQT approach, the packet consists of partial waves and de Broglie wave appears as a side product during the movement and evolution of that partial wave packet.

Since we intend to describe physical reality by a continuous field, neither the notion of particles as invariable material points nor the notion of movement can have a fundamental meaning. Only a limited zone of space wherein the quantum field strength or energy density is especially large can be considered as a particle.

In the standard quantum theory, a micro-particle is described with the help of a wave function with a probabilistic interpretation. This does not follow from the strict mathematical formalism of the non-relativistic quantum theory, but is simply postulated. A particle is represented as a point that is the source of a field, but cannot be reduced to the field itself and nothing can be said about its "structure" except with these vague words. Modern quantum field theory cannot even formulate the problem of mass spectrum searching.

This dualism is absolutely inadequate because both substances is introduced, i.e. the points and the fields. Presence of both points and fields at the same time is impossible from general philosophical positions – “razors of Ockama”. Besides that, the presence of the points leads to non-convergences, which are eliminated by various methods, including the introduction of a re-normalization group that is declined by many mathematicians and physicists, for example, P. A. M. Dirac.

The original idea of Schroedinger was to represent a particle as a wave packet of de Broglie waves. As he wrote in one of his letters, he “was happy for three months” before British mathematician Darwin showed that such packet quickly and steadily dissipates and disappears. So, it turned out that this beautiful and unique idea to represent a particle as a portion of a field was nonrealistic in the context of wave packets of de Broglie waves. Later, de Broglie tried to save this idea, he tried to prove nonlinearity till the end of his life, but he couldn’t obtain any significant result. V. E. Lyamov and L. G. Sapogin in 1968 proved [202] that every wave packet constructed from de Broglie waves with the spectrum $a(k)$ satisfying the condition of Viner-Pely (the condition for the existence of localized wave packets) became blurred in every case.

$$\int_{-\infty}^{\infty} \frac{\left| ln\left(a(k)\right) \right|}{1+k^{2}} \geq 0$$

Let us conduct the following thought experiment: at the origin of a fixed coordinate system located in an empty space free of other fields, there is a hypothetical immovable observer, past whom a particle moves along the x axis at a velocity of v<<c. Let us assume that the particle is represented by a wave packet creating a certain hitherto unknown field, and that the observer with the help of a hypothetical microprobe is measuring certain characteristics of the particle’s field at different moments in time. This measuring is done on the assumption that the size of the hypothetical energy measuring device is many times less than the size of

the particle and that it does not disrupt or influence the field created by this particle.

It is obvious that such an experiment is imaginary and cannot in principle be performed, but it doesn't prevent our imaginary device from being ideologically the simplest possible. In other words, we are interested in how the particle behaves and how it is structured when "no one is looking at it." Let the result of measurements at a certain point be function f(t), describing the structure of the wave packet, the size of which is very small and compared to the de Broglie wave. Knowing the particle's velocity v and the structural function f(t), the immovable observer can calculate the "apparent size" of the particle.

Let us assume that inside the corresponding wave packet the linearity of laws is not broken, and that the function f(t) satisfies the Dirichlet conditions and can be split into harmonic components which we will call 'partial waves'. In using the complex form of development, we can obtain:

$$f(t)=\sum_{s=-\infty}^{\infty} c_s exp(i\omega_s t), \tag{1.2.1}$$

where coefficients c_s are the amplitudes of the partial harmonics (with the mean value of $c_0 = 0$), and ω_s are the corresponding frequencies. To find the dispersion equation for partial waves, let us use the Rayleigh ratio for the group velocity v of the wave packet:

$$v = v_p + k\frac{dv_p}{dk} \tag{1.2.2}$$

Regarding the wave number k of the partial wave as a function of the phase velocity v_p, let us integrate (1.2.2) with v=const, since by the law of inertia the centre of the packet is moving at a constant speed. We will have:

$$k = \frac{C}{|v_p - v|}, \tag{1.2.3}$$

where C is the constant of integration. Integration is made on the assumption that velocity v is constant and does not depend on the frequency of the partial waves, which follows from the experimentally derived law of inertia. If we assume that the particle is a wave packet, then its group velocity will be equal to the classical velocity of the particle. Since the particle is moving at a constant speed (inertially) in the absence of external fields, the group velocity of the packet is a constant value independent of the phase velocities of the harmonic components.

The unsatisfying form of the dispersion equation (1.2.3) masks the linear dispersion law, which can be derived from (1.2.3) by substitution of $v_p = \frac{\omega_s}{k_s}$, whereby:

$$\omega_s = vk_s \pm C, \tag{1.2.4}$$

where plus sign corresponds to $v_p > v$ and minus sign corresponds to $v_p < v$.

We will now define the integration constant C as follows: since harmonic components $c_s exp(i\omega_s t)$ are propagated in the linear medium independently of each other, the behaviour of the wave packet can be presented as a superposition of the harmonic components:

$$c_s exp(i(\omega_s t - k_s x) + i\varphi) \tag{1.2.5}$$

Since the wave phase is now defined up to the additive constant, an additional constant ϕ for all partial waves is introduced. Essentially, this is possible by simple translation of the origin of the coordinates, so the value ϕ can actually be excluded from further consideration. Then, the moving wave packet can be represented as follows:

$$\Phi(x,t) = 2\Re e\sum_{1}^{\infty} c_s \exp(i(\omega_s t - k_s x)) \qquad (1.2.6)$$

Considering the wave number as a frequency function $k(\omega)$ and substituting (1.2.4) into (1.2.6), we obtain:

$$\Phi(x,t) = 2\Re e(\exp(-i(\frac{C}{v}x))\sum_{1}^{\infty} c_s \exp(i\omega_s(t - \frac{x}{v})))$$

or

$$\Phi(x,t) = \cos(\frac{C}{v}x) f(t - \frac{x}{v}) + \sin(\frac{C}{v}x) f^*(t - \frac{x}{v}), \qquad (1.2.7)$$

where function $f^*(t - \frac{x}{v})$ describes some additional partial waves with the same frequencies ω_s.

Analyzing expression (1.2.7), we can see that the wave packet $\Phi(x,t)$ in its movement in a "medium" with linear dispersion described by equation (1.2.4) is disappearing and appearing again with period $\frac{2\pi v}{C}$ in x and can be considered as being inscribed in the flat envelope modulating with that period. [The state of the wave packet (and of its corresponding particle) in the range where it disappears or its amplitude becomes very small may be thought of as a "phantom state".]

Let us find integration constant C. For this, we require the wavelength of the monochromatic envelope to be equal to the de Broglie wavelength:

$$\lambda_B = \frac{2\pi}{k_B} = \frac{2\pi v}{C} \tag{1.2.8}$$

Then, $C = vk_B$, and expression (1.2.7) become as follows:

$$\Phi(x,t) = cos(k_B x) f(t - \frac{x}{v}) + sin(k_B x) f^*(t - \frac{x}{v}). \tag{1.2.9}$$

The disappearance and reappearance of the particle occurs periodically without change of its apparent dimensions (width and form). It is clear that the dimensions of each packet can be many times less than the de Broglie wavelength.

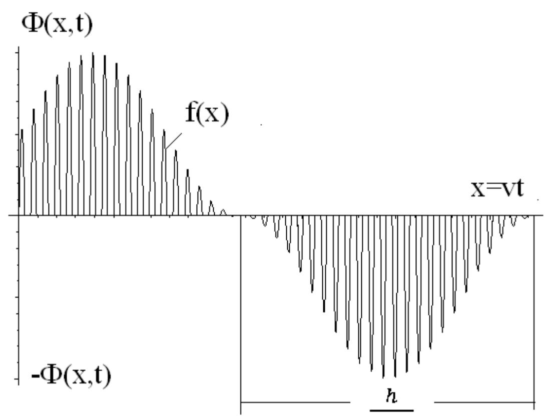

Fig. 1.2.1 *Behaviour of wave packet in linear dispersion medium (i.e., rather like a series of stroboscopic photographs).*

An approximate picture of the behaviour of such a packet in space and time [200, 201] is presented in Fig. 1.2.1 below, and the results of the mathematical modelling of the scalar Gauss wave packet behaviour in a medium with linear dispersion are presented in Fig. 1.2.2. The both figures show how such a packet disappears and reappears, changing it sign.

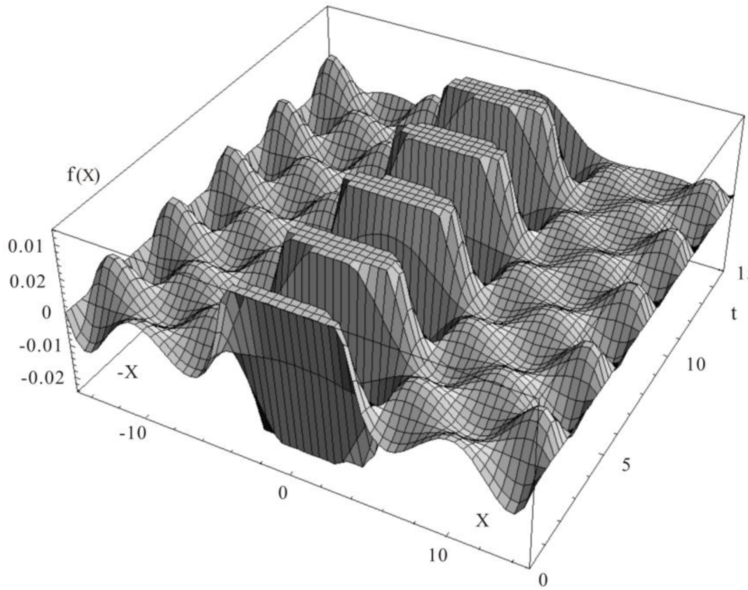

Fig. 1.2.2 *Mathematical modelling of Gauss packet behavior.*

Any dispersion without dissipation leaves the packet's energetic spectrum is unchanged. When the wave packet moves, only the phase relations between the harmonic components are changing, because the dissipation is absent. This concept is based on two postulates:

1. a particle represents a wave packet with linear field laws. The linear dispersion law follows from the law of inertia, and the particle is regarded as a moving wave packet inscribed in a flat envelope;
2. the envelope wavelength is equal to the de Broglie wavelength. Nevertheless, any packet of de Broglie waves that are localized enough is spread over the whole volume, as dispersion of the de Broglie wave $\omega_B = \dfrac{\hbar k_B^2}{2m}$ differs from linear dispersion. This does not contradict the suggested concept, as the envelope doesn't exist as a real wave and is not

included in the set of waves described by Eq. (1.2.5). More about this in section 2.13.

It is interesting to note that a dispersion space communication system has been developed, in which the transmitter emits a very long frequency-modulated impulse that cannot be detected even at a short distance, for the signal energy is widely distributed across the spectrum, yet in the transmitter area the signal turns into a short but very powerful impulse [15]. This can be achieved because the back part of the signal impulse spreads more quickly than the front, and is compressed into a very narrow impulse of the delta function type. Some species of bats use this effect in the ultrasonic range for echolocation [15]. With some imagination, it could be suggested that they learned this skill from quantum mechanics!

Please note that the process of periodicity in the appearance and disappearance of the wave-packet/particle is possible only for very small objects, and that the quantum teleportation of macro-objects being widely discussed today is hardly possible by the principles under discussion here.

However, the theoretical possibility of the wave packet spreading in the transverse direction due to diffraction is still a concern. It is in principle can be possible that the packet can disperse and not exist as a localized formation. To show that this won't happen, let us put the equation of dispersion (1.2.3) into another form. Viz., according to P. Ehrenfest, the theoretical envelope velocity of the wave packet equals the classical particle velocity:

$$v = \frac{d\omega}{dk} = \frac{P}{m} \tag{1.2.10}$$

On the other hand,

$$\omega = \frac{E}{\hbar}\text{, and } \frac{d\omega}{dk} = \frac{1}{\hbar}\frac{dE}{dk}.$$

According to classical mechanics, the energy of a free particle is:

$$E = \frac{P^2}{2m} \text{ or } \frac{d\omega}{dk} = \frac{P}{\hbar m}\frac{dP}{dk}. \tag{1.2.11}$$

Comparing (1.2.10) and (1.2.11) we obtain:

$$\frac{P}{\hbar m}\frac{dP}{dk} = \frac{P}{m},$$

and by integrating that differential equation we get

$$P = \hbar k + C.$$

Now, the phase velocity of the waves,

$$v_p = \frac{\hbar\omega_s}{\hbar k_s + C},$$

does not remain a constant value but depends on constant of integration C.

By using another method to determine the velocity phase, the constant of integration may be added to the expression of energy (but this isn't a matter of principle). The choice of the constant of integration C does not influence the results to be obtained in terms of quantum mechanics, and so for simplicity we assume that C = 0.

The present conclusion represents a known fact that motion equation invariance regarding gradient calibrating transformation. The same relations for the phase velocity of quasi-particles also hold in solid-state physics, for quasi-particle momentum that can be written as a constant divisible by the reciprocal lattice constant.

Lets return to (1.2.3):

$$k = \frac{C}{|v_p - v|}.$$

The choice of constant C determines the type of dispersion. In the general case, that relation describes the wave set with different k and λ. As we could see previously (and as is true in all inertial coordinate systems), with a certain type of dispersion the envelope of the de Broglie wave is processed in a ‘space-hold’conditions.

Putting vp = 0 in (1.2.3), we obtain

$$C = kv = \frac{mv^2}{\hbar}.$$

Substituting the value for C into the same expression (1.2.3) and taking into account that $k = \frac{\omega_s}{v_p}$, we will obtain the expression for subwave phase velocity:

$$v_p = \frac{\hbar\omega_s}{\pm mv + \frac{\hbar\omega_s}{v}}. \tag{1.2.12}$$

We should note that according to some works in quantum field theory, divergences are in principle eliminated by choice of C.

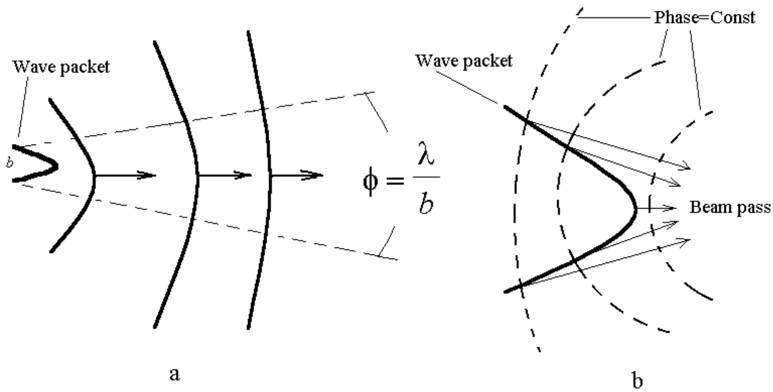

Fig. 1.2.3 *Wave packet dispersion and refocusing.*

If the theory of wave transmission is linear, the wave packet will diverge at the angle $\phi = \frac{\lambda}{b}$ (Fig. 1.2.3a).

Within the non-linear interpretation, one can see that self-focusing is able to compensate transverse diffraction (1.2.3b). For that to occur, the following relationship is necessary:

$$v_p = \frac{c}{n} = \frac{c}{n_0 + n_2 E^2},$$

where c is light velocity. Then, the peripheral phase fronts bend toward the packet's axis, thus compensating transverse diffraction [as in Fig. 1.2.3b above]. As the wave packet's mass is proportional to the square of its amplitude, relation (1.2.12) can be rewritten in the following form:

$$v_p = \frac{\hbar\omega_s}{\pm mv + \frac{\hbar\omega_s}{v}} = \frac{c}{\frac{c}{v} \pm \frac{mvc}{\hbar\omega_s}} = \frac{c}{n_0 + n_2 E^2}$$

providing $n_0 = \frac{c}{v}$, $n_2 = \pm \frac{vc}{\hbar \omega_S}$, and $m \approx E^2$ (to be discussed further).

As well we have said nothing about the nature of either the 'medium' or the waves propagating in it. In spite of various modern versions of quantum field theory, and the further development of UQT theory in the next chapters of the book, it is impossible to answer at present the very simple question "what is space-time?" Is it simply the "stage" where performers in the form of a multi-component field are continually appearing and disappearing? Or does the field represent dynamic distortions of the stage itself, so that it's impossible to separate the performers from that stage?

The authors can add little to W. Clifford's deep remarks in the epigraph of the next chapter. Here, we would like simply to remind the reader that in ordinary electrodynamics scalar and vector A conduct themselves in Lorentz transforms in the same way as time and three components of space. Moreover, while moving in space they are related to velocity v by a most natural relation:

$$\boldsymbol{A} = \boldsymbol{v}\varphi$$

The other question also appears: when the problem of corpuscular-wave dualism arose, then Schroedinger immediately arrived to idea that particle was a simple packet of de Broglie waves. Later because of packet spreading all over the space that idea was rejected although Louis de Broglie tried during all his life to make these processes non-linear and thus to save that perfect idea of the wave packet (double solutions theory and pilot wave). Why researchers have been taken no notice the extremely simple possibility of wave packet periodic disappearance and reappearance from the other wave theory discussed previously? We'll try to answer that 'philosophical' question below.

It is extremely difficult to imagine in the abstract things that never have been seen or heard before. In that light, the notion that our thoughts may merely be the reflections and transformations of something we have previously observed may be profoundly true. This may be illustrated in the following way: Relation (1.2.12) describes dispersion, including the phase velocity dependence of the wave on frequency. Then we may write the standard equation for the phase velocity

$$v_p = \frac{c}{n}$$

where c is the velocity of light and n is the index of refraction.

This coefficient, in accordance with modern theory and for any existing medium, is a complex analytical function and has both real and imaginary parts. Without any detailed assumptions about the medium, and using only the fact that any distortion will not propagate with a velocity higher than the velocity of light, Kramers and Kronig obtained the relation between real and imaginary parts of the index of refraction:

$$Re\left|n(\omega) - n(0)\right| = P\int_0^\infty \frac{2\omega^2 \, Im\, n(\omega')}{\pi\omega'(\omega'^2 - \omega^2)} d\omega'$$

Here, Re *n(ω)*is the real component of that index, If *n(ω)* is the imaginary part determining its dissipation, P indicates that the principal value of the integral is considered. From a mathematical viewpoint, this is simply a consequence of the well-known Hilbert transformation for an analytical function that does not contain poles and zeroes on the right half plane. The Cauchy integral of such function equals zero, and determines the relation between these two parts of the refraction index: that relation represents a mathematical expression of the causality principle that any medium existing in nature and consisting of atoms and molecules must satisfy.

However, in the case of the linear dispersion medium considered here, the index of refraction's imaginary part equals zero (assuming no dissipation) and so it real part is equal to zero also. The real part is then an exponential function having an imaginary index (corresponding to oscillation) and the Cauchy integral does not vanish. So, the Kramers-Kronig dispersion relations simply are not valid.

The process of periodic disappearance and reappearance of the wave packets in any real nonlinear medium consisting of particles thus cannot occur – so we could never detect it – thereby reinforcing the idea that space-time itself can hardly be considered discontinuous.

1.3 Unitary Quantum Theory

"I have no doubts about the following: small parts of space are similar in their nature to irregularities on a surface which, on the average, is flat. The quality of being curved and deformed continuously passes from one part of space to another like the phenomenon that we call the movement of matter, ethereal or corporeal. In the real physical world nothing happens except these variations, which is probably in compliance with the continuity law."

William Clifford, 1870

The wave function of a single particle (1.2.9) was derived on an assumption of non-relativistic velocities, i.e., for v<<c. To obtain its relativistic generalization it is first necessary to make the wave function relativistically phase invariant [1-3], i.e.,

$$\boldsymbol{\Phi} = exp[-i(Et - \mathbf{Px})]\mathbf{f}(\mathbf{x} - \mathbf{v}t)\,, \tag{1.3.1}$$

where

$$\mathrm{E}=\frac{\mathrm{m}}{\gamma};\mathbf{P}=\frac{\mathrm{m}\mathbf{v}}{\gamma};\gamma=\sqrt{1-\mathbf{v}^2}$$

and $f(x-vt)$ is some structural function (in this paragraph, we use a unit system in which $c=\hbar=1$). It can be required that structural function $\mathrm{f}(x-vt)$ to be scalar and satisfy the Klein-Gordon equation. Then, we will get the following equation for f:

$$\left(\mathrm{v}_i\mathrm{v}_k-\delta_{ik}\right)\frac{\partial^2\mathbf{f}}{\partial\xi_i\partial\xi_k}=0$$

Here, $\xi_i=x_i-v_it$; i, k=1, 2, 3, and summarization is obtained by repeated indices as usual. A two-component solution of the Klein-Gordon equation will then appear as follows:

$$\Phi=exp\left(-i(Et-\mathbf{Px})\right)\begin{pmatrix}\dfrac{\gamma-1}{2\gamma}\mathbf{f}-\dfrac{i}{2m}\mathbf{v}\dfrac{\partial\mathbf{f}}{\partial\xi}\\[2ex]\dfrac{\gamma+1}{2\gamma}\mathbf{f}+\dfrac{i}{2m}\mathbf{v}\dfrac{\partial\mathbf{f}}{\partial\xi}\end{pmatrix}\qquad(1.3.2)$$

By substituting (1.3.1) into the Schrödinger equation we may obtain the Laplace equation for structural function:

$$\nabla_\xi^2 f=0,$$

and it solution enable us to regard the particle as a spherical wave packet “cut into parts” by spherical harmonics.

But such approach can only serve as a certain illustration, a first approximation based on the assumption of field law linearity. Function f described by the

Laplace equation will tend to infinity at zero, which is completely unsatisfactory from the physical point of view. Let us do otherwise, and consider just the simplest equations of first and second order, which are satisfied by a one-component relativistic wave function having an arbitrary structural function.

These equations have a clearly relativistic form:

$$(u_\mu \frac{\partial}{\partial x_\mu} + im)\boldsymbol{\Phi} = 0 \quad 1.3.3)$$

$$(u_\mu u_\nu \frac{\partial^2}{\partial x_\mu \partial x_\nu} + m^2)\boldsymbol{\Phi} = 0 \quad (1.3.4)$$

where: $x_\mu = (x, it)$; $u_\mu = (\frac{\mathbf{v}}{\gamma}, \frac{i}{\gamma})$ is the particle's four-velocity; and $\mu, \vartheta = 1, 2, 3, 4$. It is natural to consider that a particle with an arbitrary spin and mass m can be described by a relativistic equation

$$(\Lambda_\mu \frac{\partial}{\partial x_\mu} + m)\boldsymbol{\Phi} = 0 \quad (1.3.5)$$

where Φ is an n-component column and Λ_μ represents four (n x 4) – matrices (n rows, 4 column) describing the spin properties of the particle. These matrices are functions of the particle velocity and satisfy relations that are defined by the spin value.

Let us now express particle energy (mass) by means of a field. For Dirac-type equations, nether the character of charge with an integer spin nor charge energy with half-integer spin are defined. In relativistic electrodynamics, according to the Laue theorem, the tensor components of the energy-impulse of the electromagnetic field that is generated by the charge do not form four-vectors, so there is only one method of expressing the particle energy:

$$E = m = \int_V \boldsymbol{\Phi}^+ \boldsymbol{\Phi} d^3 x \tag{1.3.6}$$

Usually in such cases it is required that the integral (1.3.6) contains the Green function (for example, see [19]). However, if we strictly follow the principles of the unitary theory, we should define the particle energy within non-relativistic limits as in expression (1.3.6).

Let us substitute the invariant relativistic expression $\langle \boldsymbol{\Phi} | \boldsymbol{\Phi} \rangle$ for $\int_V \boldsymbol{\Phi}^+ \boldsymbol{\Phi} d^3 x$, which, for example, equals [16] for a spin field with a rest mass differing from zero (there are also formulas for the scalar and vector fields):

$$\langle \boldsymbol{\Phi} | \boldsymbol{\Phi} \rangle = \int \{ \boldsymbol{\Phi}^* i\gamma_4 \frac{\partial}{\partial t} \hat{\varepsilon} \boldsymbol{\Phi} - \frac{\partial}{\partial t} \boldsymbol{\Phi}^* i\gamma_4 \hat{\varepsilon} \boldsymbol{\Phi} \} dV \tag{1.3.7}$$

where γ_4 is a Dirac matrix, $\hat{\varepsilon} = +1$ for a particle, and $\hat{\varepsilon} = -1$ for an antiparticle. Then, Eq. (1.3.5) will look as follows:

$$\{ \boldsymbol{\Lambda}_\mu \frac{\partial}{\partial x_\mu} + \langle \boldsymbol{\Phi} | \boldsymbol{\Phi} \rangle \} \boldsymbol{\Phi} = 0 \tag{1.3.8}$$

This nonlinear integro-differential equations are, in our view, fundamental, and must describe all the properties and interactions of particles. The mass spectrum from such equations may be derived after solving stability problems of the Sturm-Liuville type, which will in turn give the particle lifetime. In the theory under consideration, the birth and decay of all particles, and all of their interactions and transformations, are consequences of wave packet splitting and mutual diffraction phenomena due to nonlinearity. The construction of solutions to that problem will plainly require some new mathematical methods.

Point-like particles may be required to simplify the solution of the preceding Eq. (1.3.8), whereby it is reduced to the main equation of nonlinear W. Heisenberg [17] theory written not in operator form but in c-numbers. To do this we should in Eq. (1.3.5) substitute $m = \Phi^+\Phi$. Then we obtain the following equation

$$(\Lambda_\mu \frac{\partial}{\partial x_\mu} + \boldsymbol{\Phi}^+\boldsymbol{\Phi})\boldsymbol{\Phi} = 0, \tag{1.3.9}$$

thus approximate particle mass spectrum has been derived [17] with help of this equation.

Let us pass from equation (1.3.5) to the equation of particle motion in an external electromagnetic field A_μ. We therefore makes a standard substitution $\frac{\partial}{\partial x_\mu} \to \frac{\partial}{\partial x_\mu} - ieA_\mu$, and Eq. (1.3.5) is transformed as follows:

$$\left(\frac{\partial}{\partial \mathrm{t}} + \mathbf{v}\frac{\partial}{\partial \mathbf{x}} - \mathrm{iL}\right)\boldsymbol{\Phi} = 0 \tag{1.3.10}$$

where L is a relativistic Lagrangian, $L = m\gamma + e\gamma U_\mu A_\mu$.

If a particle is located in an external electromagnetic field, for example, with vector potential A and scalar potential φ, then the linear dispersion law is not changed. L and v will be certain functions of coordinates and the solution of Eq. (1.3.10) in a general form has the following form:

$$\boldsymbol{\Phi} = exp(-i\int Ldt)\mathbf{f}\left(\mathbf{x} - \int \mathbf{v}dt\right) \tag{1.3.11}$$

It is easy to make a standard transition from the relativistic case to the non-relativistic case by using the well-known transformation $\boldsymbol{\Phi} = \boldsymbol{\Phi}e^{-imt}$.

Substitution of function (1.3.11) into the equation (1.3.10) shows that the equation is satisfied providing L as a non-relativistic Lagrangian.

Let us now look at the role of the wave function phase, which is the classic action S and will enable us to establish a connection between the proposed theory and classical mechanics. Actually, the wave function may be represented in the form below (following Hamilton's principle in classic mechanics):

$$\mathbf{\Phi} = \exp(i\,S)\mathbf{f}(\mathbf{x} - \int \mathbf{v}dt)$$

If we substitute this expression into Eq. (1.3.10), we then obtain an equation for S:

$$\frac{\partial S}{dt} + \mathbf{v}\nabla S - L = 0 \qquad (1.3.12)$$

In keeping with the requirements of the Hamilton-Jacobi theory, it is necessary to assume that

$$\mathbf{P} = \nabla S;$$

then Eq. (1.3.12) will be transformed to the Hamilton-Jacobi equation:

$$\frac{\partial S}{\partial t} + H = 0,$$

where

$$H = \mathbf{Pv} - L$$

is the particle's Hamiltonian.

The function S can thereby be found, dependent on the particle's coordinates, the physical parameters of the Hamiltonian, and on q non-additive integration constants; and then perhaps the problems of motion and dynamics can be solved.

The imposed requirement $\mathbf{P} = \nabla S$ implies a transposition to classic mechanics using an optic analogy approximation, whereby the concept of particle trajectory as a beam can be introduced. Such a trajectory will be orthogonal to any given surface of a permanent operation or phase.

On the other hand, a quantum object becomes a classical construct after superposition of a large number of wave packets. The case where all wave packets composing an object spread and reintegrate simultaneously despite different velocities and phases is physically impossible. That is why such a combination when averaged out will appear, in general, like a stable and unchanging object moving under the laws of classical mechanics, whereas every elementary object obeys the quantum laws.

Note that a transfer from the unitary quantum theory to classical mechanics is mathematically strict. In the usual quantum theory, the transfer happens with an imposed condition $\hbar \to 0$. Mathematically, it is completely unsatisfactory, since $\hbar$ is some physical constant (equal to 1 if given a corresponding units system). We do not remember a single case in mathematics when a similar condition would be imposed in a proof, such as $\pi \to 0$.

Let us consider briefly the hydrogen atom problem. The solution of classical problem of particle movement in the central field allows to present the wave function (1.3.1) as follows:

$$\boldsymbol{\Phi} = e^{-iEt} e^{i\int_{r_0}^{r} p_r dr} e^{i\int_{\varphi_0}^{\varphi} p_\varphi d\varphi} f\left(r - \int_0^t v_r dt; \varphi - \int_0^t \varphi dt\right)$$

Here, r_0 and ϕ_0 are particle coordinate values (radius and angle correspondingly) at time t=0. Stationary orbits appear when the envelope is a standing wave provided:

$$ET = 2\pi n_1 h; \oint p_r dr = 2\pi n_2 h; \oint p_\phi d\phi = 2\pi n_3 h,$$

where n_1, n_2, n_3 are integers. These requirements correspond to the terms of Bohr-Sommerfeld quantification.

An integral along trajectories can be constructed with the help of (1.3.11), and it is equivalent to a flow of monochromatic particles with equally distributed phases. After developing this integral in series (see Ref. [18]), we obtain the Schroedinger equation. On the other hand, the connection of the developed approach with the Schrödinger equation follows directly from (1.3.11). The process envelope can be identified with de Broglie wave and in essence the Schroedinger equation describes the envelope of the wave packet's maxima in motion.

In conclusion of this section, let us find matrices Λ_μ. Let us assume that matrices Λ_μ are linear relative to velocity:

$$\Lambda_\mu = \Lambda_{\mu 0} + \Lambda_{\mu\nu} u_\nu \tag{1.3.13}$$

where $\Lambda_{\mu 0}$ x $\Lambda_{\mu\nu}$ are numerical matrices. Let us apply equation (1.3.5) on the left with operator $\Lambda_\sigma \frac{\partial}{\partial x_\sigma} - m$, obtaining:

$$\{\frac{1}{2}(\Lambda_\mu \Lambda_\sigma + \Lambda_\sigma \Lambda_\mu)\frac{\partial^2}{\partial x_\mu \partial x_\sigma} - m^2\}\boldsymbol{\Phi} = 0 \tag{1.3.14}$$

If we require that each component of system (1.3.14) satisfies the second order equation (1.3.4), and then

$$\Lambda_\mu \Lambda_\sigma + \Lambda_\sigma \Lambda_\mu = -2u_\mu u_\sigma I \tag{1.3.15}$$

Relation (1.3.15) is satisfied identically if we take ten Hermitian matrices 32x32 as numerical matrices $\Lambda_{\mu\nu}$, satisfying the following commutation relations:

$$\Lambda_{\mu\nu}\Lambda_{\sigma\tau} + \Lambda_{\sigma\tau}\Lambda_{\mu\nu} = 2(\delta_{\mu\sigma}\delta_{\nu\tau} - \delta_{\mu\tau}\delta_{\nu\sigma})I \tag{1.3.16}$$

Here, indices μ, ν, σ, τ take values 0, 1, 2, 3, 4. It is interesting to note that if the particle's 4-velocity is assumed to be zero (u_μ=0), then system (1.3.5) will reduce to eight similar Dirac equations.

However, this requirement is absolutely unsatisfactory both from the physical and the mathematical points of view. Four-velocity has 4 components, three of them are usual components of the particle velocity along three axes, and they really can tend to zero. But the same impossible for the fourth component.

Hence, this approach is formally incorrect and requires explanation. In our view, although the Dirac equation describes the hydrogen atom spectrum absolutely correctly, it is not properly a fundamental equation. It has two weak points:

1) the correct magnitude of the velocity operator's proper value is absent. It is known that in any problem of this type the proper value of the velocity operator is always equal to the velocity of light! In fact, Russian physicist and mathematician V. A. Fok regarded this as an essential defect of the Dirac theory;

2) The Klein paradox [19] appears in the solution of the problem of barrier passage, when the number of the particles that pass is bigger than the number of incident particles.

The equations of the Unitary Quantum Theory we are proposing are more correct and fundamental. For this reason, a transition from correct fundamental equations to the incompletely accurate Dirac equation needs such a strange requirement as $u_{\mu} = 0$.

1.4 Relativistic Invariance, Commutation Relations and Deriving the Value of the Fine Structure Constant

Everything went very well, until the Austrian General Headquarters interfered: the shells were taken to the rear, and the wounded to the front.

Jaroslav Hasek, The Good Soldier Schweik

The previous investigations [2, 3, 200, 201] have suggested a model of the unitary field theory where a particle with mass m is described by the equation

$$i\lambda^{\mu}\frac{\partial \boldsymbol{\Phi}}{\partial x^{\mu}} - m\boldsymbol{\Phi} = 0 \tag{1.4.1}$$

and each component $\boldsymbol{\Phi}_s$ of the wave function satisfies the second order equation

$$u^{\mu}u^{\nu}\frac{\partial^2 \boldsymbol{\Phi}_s}{\partial x^{\mu}\partial x^{\nu}} + m^2\boldsymbol{\Phi}_s = 0\,, \tag{1.4.2}$$

so that the commutation relations for matrices λ^{μ} have the form

$$\lambda^{\mu}\lambda^{\nu} + \lambda^{\nu}\lambda^{\mu} = 2g^{\mu\nu}I \tag{1.4.3}$$

where $\boldsymbol{x}^{\mu} = (t, \mathbf{x}); u^{\mu} = (\frac{1}{\gamma}, \frac{\mathbf{v}}{\gamma})$ is the particle velocity; $\mu, \boldsymbol{\nu} = 0,1,2,3$; a metrics with signature (+,-,-,-) is used; c and h equal 1, and repeated indices are assumed to be summed.

1. The commutation relation.

For equation (1.4.1) to be the starting point of the theory, the equation should first result in the correct energy-momentum relation for a free particle and then be the Lorentz covariant. Equation (1.4.2) meets the former condition in the form

$$\left(p^{\mu}u_{\mu}\right)^{2}=m^{2}$$

Matrices are functions of the particle velocity, and thus the commutation relations (1.4.3) alone are insufficient for proving invariance of Eq. (1.4.1) under the Lorentz transformations; therefore let us first specify the functional dependence of the matrices on the velocity. Since the trivial solution

$$\lambda^{\mu}=u^{\mu}I$$

is totally uninteresting, let us consider the case of linear dependence on the velocity

$$\lambda^{\mu}=\lambda^{\mu\sigma}u_{\sigma}+\lambda^{\mu4} \tag{1.4.4}$$

where $\lambda^{\mu\sigma}$ and $\lambda^{\mu4}$ are numerical matrices. The condition (3) holds identically if

$$\begin{aligned}
&\lambda^{\mu\sigma}\lambda^{\nu\tau}+\lambda^{\nu\tau}\lambda^{\mu\sigma}=2\left(g^{\mu\tau}g^{\nu\sigma}-g^{\mu\sigma}g^{\nu\tau}\right)I\\
&\lambda^{\mu4}\lambda^{\nu4}+\lambda^{\nu4}\lambda^{\mu4}=2g^{\mu\nu}I\\
&\lambda^{\mu4}\lambda^{\nu\tau}+\lambda^{\nu\tau}\lambda^{\mu4}=0
\end{aligned} \tag{1.4.5}$$

Because of the antisymmetry of $\lambda^{\mu\sigma}=-\lambda^{\sigma\mu}$, only ten out of the twenty matrices are independent quantities. These matrices mutually anticommute, the square of four of them is equal to unity and that of six, to minus unity. To put it differently, Eq. (1.4.5) is specified by ten generatrices of the alternion algebra

${}^{4}A_{11}$, which is isomorphous with the algebra of the sixteenth order quaternion matrices [23]. Since they are not convenient, let us replace the quaternion matrices with ten complex, irreducible, unitary 32nd order matrices

$$\left(\lambda^{\mu\nu}\right)^{+} = \left(\lambda^{\mu\nu}\right)^{-1}, \quad \left(\lambda^{\mu 4}\right)^{+} = \left(\lambda^{\mu 4}\right)^{-1} \tag{1.4.6}$$

This situation arises in construction of Dirac matrices, which are usually chosen as complex fourth order matrices even though the equation

$$\gamma^{\mu}\gamma^{\nu} + \gamma^{\nu}\gamma^{\mu} = 2g^{\mu\nu}I$$

is satisfied by four second–order quaternion matrices.

From eqs. (1.4.5) and (1.4.6) it follows that four matrices are Hermitian and six are anti-Hermitian

$$\left(\lambda^{0a}\right)^{+} = \lambda^{0a}, \quad \left(\lambda^{ab}\right)^{+} = -\lambda^{ab}, \text{ a, b=1, 2, 3, 4} \tag{1.4.7}$$

If a matrix Λ is introduced

$$\Lambda = \lambda^{12}\lambda^{13}\lambda^{14}\lambda^{23}\lambda^{24}\lambda^{34}, \quad \Lambda^{+} = \Lambda^{-1} = -\Lambda \tag{1.4.8}$$

then the Hermitian conjugations conditions (7) can be rearranged into

$$\left(\lambda^{\alpha\beta}\right)^{+} = \Lambda\lambda^{\alpha\beta}\Lambda^{-1} \tag{1.4.9}$$

Represented in the form (1.4.5) the commutation relations are unwieldy and inconvenient in proving the relativistic invariance; however, they can be represented in a simpler form. Let us define a symmetrical tensor $g_{\alpha\beta}$

$$g_{00} = -g_{11} = -g_{22} = -g_{33} = -g_{44} = 1 \quad g_{\alpha\beta} = 0 \text{ if } \alpha \neq \beta \tag{1.4.10}$$

henceforth subscripts of initial letters of the Greek alphabet $\alpha, \beta, \gamma, \delta$ take on values from 0 to 4 while those of the middle of the alphabet from 0 to 3. The inverse tensor $g^{\alpha\beta}$ provides a compact restatement of commutation relation (1.4.5)

$$\lambda^{\alpha\beta}\lambda^{\gamma\delta} + \lambda^{\gamma\delta}\lambda^{\alpha\beta} = 2\left(g^{\alpha\delta}g^{\beta\gamma} - g^{\alpha\gamma}g^{\beta\delta}\right)I \tag{1.4.11}$$

Eqs. (1.4.4), (1.4.10) and (1.4.11) make it possible to prove the relativistic invariance of Eq. (1.4.1) by using a five-dimensional group of transformations of coordinate O (4, 1). For this purpose extend Eq. (1.4.1) to the case of a five-dimensional pseudo-Euclidian space with a metric tensor (1.4.10)

$$i\lambda^{\alpha\beta}u_{\alpha}\frac{\partial \boldsymbol{\Phi}}{\partial x^{\beta}} - m\boldsymbol{\Phi} = 0 \tag{1.4.12}$$

(where u^{α} is the 5-velocity, $u^{\alpha}u_{\alpha} = 0$) and then prove invariance of this equation under the group of five-dimensional transformation O (4, 1), which contains the Lorentz group as a subgroup. Under reduction of O (4, 1) to the Lorentz group, we assume that $x^4 = Const, u^4 = 1$ and $\frac{\partial}{\partial x^4} \equiv 1$ then we have Eq. (1); in other words, one can assume that Eq. (1.4.1) is invariant under five-dimensional transformations, but the physical solution does not depend on the fifth coordinate. Incidentally, Eq. (1.4.12) can be interpreted differently, but we will not discuss these possibilities, for using the five dimensions is merely a convenient tool, which enables us to make full use of simplicity of the commutation relations (1.4.11).

2. The invariance of the Equation.

To prove invariance of the equation, it is sufficient to show [23] that for any transformation of coordinates

$$\left(x^{\alpha}\right)' = a^{\alpha}_{\beta} x^{\beta}\,;\;\; \left(x^{\alpha}\right)' x'_{\alpha} = inv \tag{1.4.13}$$

there is a linear transformation S(a) of wave functions, the primed and unprimed reference frame

$$\boldsymbol{\Phi}'\left(x'\right) = S\left(a\right)\boldsymbol{\Phi}\left(x\right);\;\; \boldsymbol{\Phi}\left(x\right) = S^{-1}\left(a\right)\boldsymbol{\Phi}'\left(x'\right) \tag{1.4.14}$$

and $\boldsymbol{\Phi}'\left(x'\right)$ is a solution of the equation, which has the form of Eq. (1.4.12) in the primed reference frame

$$\left[i\lambda^{\gamma\delta}u'_{\gamma}\frac{\partial}{\partial\left(x^{\delta}\right)'} - m\right]\boldsymbol{\Phi}'\left(x'\right) = 0 \tag{1.4.15}$$

Substitute (1.4.14) into (1.4.12); multiply the left-hand side by S(a), and use the definition (1.4.13) to have

$$\left[iS\lambda^{\alpha\beta}S^{-1}a^{\gamma}_{\alpha}a^{\delta}_{\beta}u'_{\gamma}\frac{\partial}{\partial\left(x^{\delta}\right)'} - m\right]\boldsymbol{\Phi}'\left(x'\right) = 0$$

This equation coincides with (1.4.15), if the matrix has the property

$$a^{\gamma}_{\alpha}a^{\delta}_{\beta}S\lambda^{\alpha\beta}S^{-1} = \lambda^{\gamma\delta} \tag{1.4.16}$$

Construct S for the infinitesimal proper transformation of the group O (4, 1)

$$a^{\beta}_{\alpha} = \delta^{\beta}_{\alpha} + \varepsilon^{\beta}_{\alpha}\,; a_{\alpha\beta} = g_{\alpha\beta} + \varepsilon_{\alpha\beta} \tag{1.4.17}$$

with

$$\varepsilon_{\alpha\beta} = -\varepsilon_{\beta\alpha} \tag{1.4.18}$$

Expand S in power of ε and keep only linear terms

$$S = 1 - \frac{1}{4}\sigma^{\alpha\beta}\varepsilon_{\alpha\beta} \tag{1.4.19}$$

where $\sigma^{\alpha\beta} = -\sigma^{\beta\alpha}$ by Eq. (1.4.18). Substitute eqs. (1.4.17)-(1.4.19) into Eq. (1.4.16), keep first-order terms in ε, use the notation [B, C]=BC-CB for the commutation brackets and have

$$2\left[\sigma^{\alpha\beta}, \lambda^{\gamma\delta}\right] = g^{\alpha\delta}\lambda^{\beta\gamma} - g^{\alpha\gamma}\lambda^{\beta\delta} + g^{\beta\gamma}\lambda^{\alpha\delta} - g^{\beta\delta}\lambda^{\alpha\gamma}$$

The antisymmetric solution of this equation

$$\sigma^{\alpha\beta} = \frac{1}{2} g_{\gamma\delta}\left[\lambda^{\beta\gamma}, \lambda^{\alpha\delta}\right] \tag{1.4.20}$$

is, by virtue of diagonality of the metric tensor and antisymmetry of $\lambda^{\alpha\beta}$, a sum of mutually commutating terms; in particular, σ^{12} has the form

$$\sigma^{12} = \lambda^{20}\lambda^{10} - \lambda^{23}\lambda^{13} - \lambda^{24}\lambda^{14}$$

According to Eq. (1.4.19) S for an infinitesimal transformation is given by

$$S = 1 - \frac{1}{8} g_{\gamma\delta}\varepsilon_{\alpha\beta}\left[\lambda^{\beta\gamma}, \lambda^{\alpha\beta}\right]$$

Hence, for rotation through a finite angle ω about this axis in the direction labelled n is represented as

$$S = \exp\left\{-\frac{1}{4}\omega\sigma^{\alpha\beta}P^{n}_{\alpha\beta}\right\} \tag{1.4.21}$$

where $P^{n}_{\alpha\beta}$ is the generator of rotation about this axis. Generally speaking the matrix S is not unitary but formula (1.4.9) easily shows that

$$\Lambda^{-1}\sigma^{+}\Lambda = -\sigma,$$

consequently, for proper transformations

$$\Lambda^{-1}S^{+}\Lambda = S^{-1} \tag{1.4.22}$$

Let us consider improper transformations of space reflection and time reversal. For space reflection the matrix a is diagonal

$$a_0^0 = a_4^4 = -a_1^1 = -a_2^2 = -a_3^3 = 1,$$

then Eq. (1.4.16) for the space reflection operator P is satisfied by

$$P = \lambda^{01}\lambda^{02}\lambda^{03}\lambda^{14}\lambda^{24}\lambda^{34} = P^{+} = P^{-1} \tag{1.4.23}$$

which ensures invariance of both Eq. (1.4.1) and Eq. (1.4.12).

Construct a transformation of the time inversion; for this purpose introduce an interaction of a particle whose charge is e with an external electromagnetic field $A^{\mu} = \left(\phi, A^{k}\right)$ by means of the gauge invariant substitution

$$i\frac{\partial}{\partial x^{\mu}} \to i\frac{\partial}{\partial x^{\mu}} - eA_{\mu}$$

and rewrite Eq. (1.4.1) in the form [2, 3, 6]:

$$i\lambda^{0}\frac{\partial \boldsymbol{\Phi}}{\partial t} = \left[\lambda^{k}\left(-i\frac{\partial}{dx^{k}} + eA_{k}\right) + m + e\phi\lambda^{0}\right]\boldsymbol{\Phi} = H\boldsymbol{\Phi}$$

Determine transformation T as such that if $t' = -t, \Phi_T' = \Phi'\left(t'\right) = T\Phi\left(t\right)$; then the latter equation becomes

$$-\left(Ti\lambda^{0}T^{-1}\right)\frac{\partial \boldsymbol{\Phi}'\left(t'\right)}{\partial t'} = \left(THT^{-1}\right)\boldsymbol{\Phi}'\left(t'\right)$$

When the sense of time is reserved

$$u'_0 = u_0; u'_k = -u_k; \boldsymbol{\Phi}' = \boldsymbol{\Phi}; A'^k = -A^k$$

and, before all, it is necessary to change the sign between two terms $i\frac{\partial}{\partial x^k}$ and eA_k; therefore the transformation is regarded as a complex conjugation operator multiplied by the matrix T:

$$\boldsymbol{\Phi}'_T = T\boldsymbol{\Phi}(t) = T\boldsymbol{\Phi}^*(t) \tag{1.4.24}$$

This gives

$$i\left(T\overset{*}{\lambda}{}^0 T^{-1}\right)\frac{\partial\boldsymbol{\Phi}'(t')}{\partial t'} = \left\{-(T\overset{*}{\lambda}{}^k T^{-1})\left[-i\frac{\partial}{\partial\left(x^k\right)'} + eA'_k\right] + m + e\phi\left(T\overset{*}{\lambda}{}^0 T^{-1}\right)\right\}\boldsymbol{\Phi}'(t')$$

and for invariance of the equation it is necessary that

$$T\overset{*}{\lambda}{}^{0k} T^{-1} = -\lambda^{0k}; T\overset{*}{\lambda}{}^{k2} T^{-1} = \lambda^{k2}; T\overset{*}{\lambda}{}^{k4} T^{-1} = -\lambda^{k4}; T\overset{*}{\lambda}{}^{04} T^{-1} = \lambda^{04} . \tag{1.4.25}$$

Thence it immediately follows that $T^* = T^{-1} = T$, though the explicit form of the matrix T depends on the particular representation of the matrix $\lambda^{\alpha\beta}$. Note that there is just one matrix

$$\lambda = \prod_{\alpha<\beta}^{4} \lambda^{\alpha\beta}$$

which commutes with both generators $\sigma^{\alpha\beta}$ for the representation of the group O (4, 1) and with the operators of discrete transformation P and T. Under reduction of O (4, 1) to the Lorentz group two more matrices

$$\Lambda_1 = \lambda^{04}\lambda^{14}\lambda^{24}\lambda^{34}; \Lambda_2 = \lambda\Lambda_1$$

are generated which commute with the generators $\sigma^{\mu\sigma}$ of the representation of the Lorentz group and anticommute with P and T. Consequently, formulae (1.4.21), (1.4.23)-(1.4.25) specify the reducible representation of the Lorentz group and this representation is double-valued. Indeed, consider a particular case, rotation through angle ω about the Z-axis. In this case $P_{12}^{Z} = -P_{21}^{Z} = 1$; using the explicit form of σ^{12} we have

$$S = exp\left(-\frac{\omega}{2}\sigma^{12}\right)$$
$$= cos^{3}\left(\frac{\omega}{2}\right) + \sigma^{12} cos^{2}\left(\frac{\omega}{2}\right) sin\left(\frac{\omega}{2}\right) + \frac{3+\left(\sigma^{12}\right)^{2}}{2} cos\left(\frac{\omega}{2}\right) sin^{2}\left(\frac{\omega}{2}\right)$$
$$+\lambda^{20}\lambda^{10}\lambda^{23}\lambda^{13}\lambda^{24}\lambda^{14} sin^{3}\left(\frac{\omega}{2}\right)$$

The half-angle is an expression of the double value of the wave function transformation. Therefore the observables in the theory should be bilinear in $\boldsymbol{\Phi}(x)$. The matrix Λ makes it possible to determine the adjoint wave function $\bar{\boldsymbol{\Phi}} = \boldsymbol{\Phi}^{+}\Lambda$, which is a solution of the adjoint equation

$$i\frac{\partial \bar{\boldsymbol{\Phi}}}{\partial x^{\mu}}\lambda^{\mu} + m\bar{\boldsymbol{\Phi}} = 0$$

An adjoint wave function under an arbitrary transformation of the co-ordinates should be transformed by the equation $\bar{\boldsymbol{\Phi}}' = \bar{\boldsymbol{\Phi}}\Lambda^{-1}S^{+}\Lambda$ which for proper rotations (1.4.22) leads to $\bar{\boldsymbol{\Phi}}' = \bar{\boldsymbol{\Phi}}S^{-1}$, for space and time inversions $\bar{\boldsymbol{\Phi}}'_{P} = -\bar{\boldsymbol{\Phi}}P$ and $\bar{\boldsymbol{\Phi}}' = -\bar{\boldsymbol{\Phi}}^{*}T^{-1}$, respectively. The adjoint wave function and the matrices λ, Λ_{1} and Λ_{2} make it possible to construct four independent scalar functions $\bar{\boldsymbol{\Phi}}\boldsymbol{\Phi};\bar{\boldsymbol{\Phi}}\lambda\boldsymbol{\Phi};\bar{\boldsymbol{\Phi}}\Lambda_1\boldsymbol{\Phi};$ and $\bar{\boldsymbol{\Phi}}\Lambda_2\boldsymbol{\Phi}$, which under space and time inversions are transformed as

$$\bar{\boldsymbol{\Phi}}'_P\boldsymbol{\Phi}'_P=-\bar{\boldsymbol{\Phi}}\boldsymbol{\Phi},\quad \bar{\boldsymbol{\Phi}}'_T\boldsymbol{\Phi}'_T=\bar{\boldsymbol{\Phi}}\boldsymbol{\Phi} \tag{1.4.26a}$$

$$\bar{\boldsymbol{\Phi}}'_P\lambda\boldsymbol{\Phi}'_P=-\bar{\boldsymbol{\Phi}}\lambda\boldsymbol{\Phi},\quad \bar{\boldsymbol{\Phi}}'_T\lambda\boldsymbol{\Phi}'_T=-\bar{\boldsymbol{\Phi}}\lambda\boldsymbol{\Phi} \tag{1.4.26b}$$

$$\bar{\boldsymbol{\Phi}}'_P\Lambda_1\boldsymbol{\Phi}'_P=\bar{\boldsymbol{\Phi}}\Lambda_1\boldsymbol{\Phi}\quad \bar{\boldsymbol{\Phi}}'_T\Lambda_1\boldsymbol{\Phi}'_T=-\bar{\boldsymbol{\Phi}}\Lambda_1\boldsymbol{\Phi} \tag{1.4.26c}$$

$$\bar{\boldsymbol{\Phi}}'_P\Lambda_2\boldsymbol{\Phi}'_P=\bar{\boldsymbol{\Phi}}\Lambda_2\boldsymbol{\Phi},\quad \bar{\boldsymbol{\Phi}}'_T\Lambda_2\boldsymbol{\Phi}'_T=\bar{\boldsymbol{\Phi}}\Lambda_2\boldsymbol{\Phi} \tag{1.4.26d}$$

Following the classification of [16, 23], the quantities (26a-d) are singular and simple pseudo-scalar and singular and simple scalar, respectively, each of these functions being a unique scalar function of the associated type, quadratic in $\boldsymbol{\Phi}(x)$. To obtain a numerical scalar let us use a representation of the function $\boldsymbol{\Phi}(x)$ as a four-dimensional Fourier integral. Since each component of $\boldsymbol{\Phi}(x)$ satisfies the second order equation (2), the general solution represented entirely in relativistic terms has the form

$$\boldsymbol{\Phi}(x)=\frac{2}{(2\pi)^{3/2}}\int d^4k\, e^{ik_\mu x^\mu}\delta\left\{\left(k_\mu u^\mu\right)^2-m^2\right\}\boldsymbol{\Phi}(k) \tag{1.4.27}$$

where

$$\delta\left\{\left(k_\mu u^\mu\right)^2-m^2\right\}=\frac{1}{2m}\left\{\delta\left(k_\mu u^\mu-m\right)+\delta\left(k_\mu u^\mu+m\right)\right\}$$

is the relativistic δ-function and the amplitude $\Phi(k)=\Phi(k^0,\mathrm{k})$ satisfies the equation

$$\left(\lambda^{\mu} k_{\mu}+m\right)\Phi(k)=0 \quad \text{for} \quad (ku)^{2}=m^{2}$$

Because the integrand includes a δ-function, the integration is performed over just two Lorentz-invariant hyper surfaces $k_{\mu}u^{\mu}=\pm m$, rather than the entire four-dimensional k-space. This allows for decomposing the integral (27) into two summands

$$\boldsymbol{\Phi}(x)=\boldsymbol{\Phi}^{+}(x)+\boldsymbol{\Phi}^{-}(x);\boldsymbol{\Phi}^{\pm}(x)=\frac{1}{(2\pi)^{3/2}}\int d^{4}k\frac{\delta\left(k_{\mu}u^{\mu}\mp m\right)}{2m}\boldsymbol{\Phi}(k) \qquad (1.4.28)$$

Using this representation and integrating over the three-dimensional volume, we have

$$\int\bar{\boldsymbol{\Phi}}^{\pm}u^{\mu}\frac{\partial\boldsymbol{\Phi}}{\partial x^{\mu}}\frac{dV}{\gamma}=-\int u^{\mu}\frac{\partial\bar{\boldsymbol{\Phi}}^{\pm}}{\partial x^{\mu}}\boldsymbol{\Phi}^{\pm}\frac{dV}{\gamma}=\pm\frac{i}{2m}\int d^{4}k\delta\left(k_{\mu}u^{\mu}\mp m\right)\bar{\boldsymbol{\Phi}}(k)\boldsymbol{\Phi}(k)$$

$$\int\bar{\boldsymbol{\Phi}}^{\pm}u^{\mu}\frac{\partial\boldsymbol{\Phi}^{\mp}}{\partial x^{\mu}}\frac{dV}{\gamma}=\int u^{\mu}\frac{\partial\bar{\boldsymbol{\Phi}}^{\pm}}{\partial x^{\mu}}\boldsymbol{\Phi}^{\mp}\frac{dV}{\gamma}=\mp\frac{i}{2m}\int dk\exp\left(\mp\frac{2ix^{0}m}{u^{0}}\right)\bar{\boldsymbol{\Phi}}\left(\frac{\mathrm{ku}\pm m}{u^{0}},\mathrm{k}\right)\boldsymbol{\Phi}\left(\frac{\mathrm{ku}\mp m}{u^{0}},\mathrm{k}\right)$$

Combining these relations and using the equality

$$\delta\left(k_{\mu}u^{\mu}-m\right)-\delta\left(k_{\mu}u^{\mu}+m\right)=\theta\left(k_{\mu}u^{\mu}\right)\delta\left\{\left(k_{\mu}u^{\mu}\right)^{2}-m^{2}\right\}$$

we find that

$$\int\left(\bar{\boldsymbol{\Phi}}u^{\mu}\frac{\partial\boldsymbol{\Phi}}{\partial x^{\mu}}-u^{\mu}\frac{\partial\bar{\boldsymbol{\Phi}}}{\partial x^{\mu}}\boldsymbol{\Phi}\right)\frac{dV}{\gamma}=i\int d^{4}k\theta\left(k_{\mu}u^{\mu}\right)\delta\left\{\left(k_{\mu}u^{\mu}\right)^{2}-m^{2}\right\}\bar{\boldsymbol{\Phi}}(k)\boldsymbol{\Phi}(k) \qquad (1.4.29)$$

where

$$\theta(ku) = \begin{cases} 1, \text{if } ku > 0 \\ \text{-1, if } ku < 0 \end{cases}$$

The right-hand side of Eq. (1.4.29) is explicitly represented in covariant form, which facilitates a study of properties, which can be traced to the space and time inversions. More specifically, Eq. (1.4.29) is a simple pseudo-scalar because $\int \cdots d^4k$ and $\delta\left\{\left(k_\mu u^\mu\right)^2 - m^2\right\}$ are simple scalars, $\theta\left(k_\mu u^\mu\right)$ is a singular scalar, (θ is an odd function and k^μ and u^μ are simple and singular vectors, respectively), and $\bar{\boldsymbol{\Phi}}(k)\boldsymbol{\Phi}(k)$ is a singular pseudo-scalar, according to the definition (1.4.27) and Eq. (1.4.26a).

3. The mass Determination

It is easy to construct a simple scalar

$$\int\left(\bar{\boldsymbol{\Phi}} \Lambda_1 u^\mu \frac{\partial \boldsymbol{\Phi}}{\partial x^\mu} - u^\mu \frac{\partial \bar{\boldsymbol{\Phi}}}{\partial x^\mu} \Lambda_1 \boldsymbol{\Phi}\right)\frac{dV}{\gamma}$$

which can, following [2, 3, 6, 23], be interpreted as the particle mass while the nonlinear equation [6] is represented as follows:

$$i\lambda^\mu \frac{\partial \boldsymbol{\Phi}}{\partial x^\mu} - \boldsymbol{\Phi}\int\left(\bar{\boldsymbol{\Phi}} \Lambda_1 u^\mu \frac{\partial \boldsymbol{\Phi}}{\partial x^\mu} - u^\mu \frac{\partial \bar{\boldsymbol{\Phi}}}{\partial x^\mu} \Lambda_1 \boldsymbol{\Phi}\right)\frac{dV}{\gamma} = 0 \qquad (1.4.30)$$

Unfortunately, the authors can only look at this fundamental (in our view) equation. It appears that any further progress in finding a solution to such an equation will be achieved with the help of computers and future symbol mathematics programs (of the Maple-18, Mathematica-9 types, etc.). For this purpose equation (1.4.30) should have a form with a clear matrix appearance. It is well known that the solution will not depend on a concrete representation of matrices λ_{μ}, Λ_2, it is only important that the commutations relations are satisfied. By the way, the latter can be checked by direct finding of commutators and anticommutators with apparent matrix representation. Let us note that the authors of [2-4] had received these results long before the epoch of personal computers and symbol math programs. When these things appeared, the first thing the authors did was to check the correctness of matrix correlations of the size 32x32!

In order to receive a concrete appearance of all the matrices, let us apply the bloc ideas. For this purpose, let us write down the basic matrices $\gamma 0, \gamma 1, \gamma 2, \gamma 3, g^{\mu\nu}, Z, i$

$$\gamma 0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} \quad \gamma 1 = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{bmatrix} \quad \gamma 2 = \begin{bmatrix} 0 & 0 & 0 & -I \\ 0 & 0 & I & 0 \\ 0 & I & 0 & 0 \\ -I & 0 & 0 & 0 \end{bmatrix} \quad Z = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\gamma 3 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \quad \gamma 4 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \quad g_{\mu\nu} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} \quad i = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

For these matrices the following standard commutation relations are correct:

$$\gamma^{\mu}\gamma^{\nu} + \gamma^{\nu}\gamma^{\mu} = 2g^{\mu\nu}; \quad \mu, \nu = 0,1,2,3;$$

where $\mu, \nu, \sigma, \tau = 0, 1, 2, 3, 4$ and $g = +,-,-,-$.

From these basic matrices 10 supplementary bloc matrices can be constructed - $\lambda^{01}, \lambda^{02}, \lambda^{03}, \lambda^{04}, \lambda^{12}, \lambda^{13}, \lambda^{14}, \lambda^{23}, \lambda^{24}, \lambda^{34}$, which have a clear appearance:

$$\Lambda_{01} = \begin{bmatrix} i & Z & Z & Z & Z & Z & Z & Z \\ Z & i & Z & Z & Z & Z & Z & Z \\ Z & Z & i & Z & Z & Z & Z & Z \\ Z & Z & Z & i & Z & Z & Z & Z \\ Z & Z & Z & Z & -i & Z & Z & Z \\ Z & Z & Z & Z & Z & -i & Z & Z \\ Z & Z & Z & Z & Z & Z & -i & Z \\ Z & Z & Z & Z & Z & Z & Z & -i \end{bmatrix}$$

$$\Lambda_{02} = \begin{bmatrix} Z & Z & Z & Z & i & Z & Z & Z \\ Z & Z & Z & Z & Z & i & Z & Z \\ Z & Z & Z & Z & Z & Z & -i & Z \\ Z & Z & Z & Z & Z & Z & Z & -i \\ i & Z & Z & Z & Z & Z & Z & Z \\ Z & i & Z & Z & Z & Z & Z & Z \\ Z & Z & -i & Z & Z & Z & Z & Z \\ Z & Z & Z & -i & Z & Z & Z & Z \end{bmatrix}$$

$$\Lambda_{03} = \begin{bmatrix} Z & Z & Z & Z & Z & Z & i & Z \\ Z & Z & Z & Z & Z & Z & Z & -i \\ Z & Z & Z & Z & i & Z & Z & Z \\ Z & Z & i & Z & Z & -i & Z & Z \\ Z & Z & Z & -i & Z & Z & Z & Z \\ Z & Z & Z & Z & Z & Z & Z & Z \\ i & Z & Z & Z & Z & Z & Z & Z \\ Z & -i & Z & Z & Z & Z & Z & Z \end{bmatrix}$$

$$\Lambda_{04} = \begin{bmatrix} Z & Z & Z & Z & Z & Z & Z & \gamma_0 \\ Z & Z & Z & Z & Z & Z & \gamma_0 & Z \\ Z & Z & Z & Z & Z & \gamma_0 & Z & Z \\ Z & Z & Z & Z & \gamma_0 & Z & Z & Z \\ Z & Z & Z & \gamma_0 & Z & Z & Z & Z \\ Z & Z & \gamma_0 & Z & Z & Z & Z & Z \\ Z & \gamma_0 & Z & Z & Z & Z & Z & Z \\ \gamma_0 & Z & Z & Z & Z & Z & Z & Z \end{bmatrix}$$

$$\Lambda_{12} = \begin{bmatrix} Z & Z & Z & Z & Z & Z & Z & \gamma_1 \\ Z & Z & Z & Z & Z & Z & \gamma_1 & Z \\ Z & Z & Z & Z & Z & \gamma_1 & Z & Z \\ Z & Z & Z & Z & \gamma_1 & Z & Z & Z \\ Z & Z & Z & \gamma_1 & Z & Z & Z & Z \\ Z & Z & \gamma_1 & Z & Z & Z & Z & Z \\ Z & \gamma_1 & Z & Z & Z & Z & Z & Z \\ \gamma_1 & Z & Z & Z & Z & Z & Z & Z \end{bmatrix}$$

$$\Lambda_{13}=\begin{bmatrix} Z & Z & Z & Z & Z & Z & Z & \gamma_2 \\ Z & Z & Z & Z & Z & Z & \gamma_2 & Z \\ Z & Z & Z & Z & Z & \gamma_2 & Z & Z \\ Z & Z & Z & Z & \gamma_2 & Z & Z & Z \\ Z & Z & Z & \gamma_2 & Z & Z & Z & Z \\ Z & Z & \gamma_2 & Z & Z & Z & Z & Z \\ Z & \gamma_2 & Z & Z & Z & Z & Z & Z \\ \gamma_2 & Z & Z & Z & Z & Z & Z & Z \end{bmatrix}$$

$$\Lambda_{14}=\begin{bmatrix} Z & Z & Z & Z & Z & Z & Z & \gamma_3 \\ Z & Z & Z & Z & Z & Z & \gamma_3 & Z \\ Z & Z & Z & Z & Z & \gamma_3 & Z & Z \\ Z & Z & Z & Z & \gamma_3 & Z & Z & Z \\ Z & Z & Z & \gamma_3 & Z & Z & Z & Z \\ Z & Z & \gamma_3 & Z & Z & Z & Z & Z \\ Z & \gamma_3 & Z & Z & Z & Z & Z & Z \\ \gamma_3 & Z & Z & Z & Z & Z & Z & Z \end{bmatrix}$$

$$\Lambda_{23}=\begin{bmatrix} Z & Z & Z & Z & Z & Z & Z & -i \\ Z & Z & Z & Z & Z & Z & i & Z \\ Z & Z & Z & Z & Z & -i & Z & Z \\ Z & Z & Z & Z & i & Z & Z & Z \\ Z & Z & Z & -i & Z & Z & Z & Z \\ Z & Z & i & Z & Z & Z & Z & Z \\ Z & -i & Z & Z & Z & Z & Z & Z \\ i & Z & Z & Z & Z & Z & Z & Z \end{bmatrix}$$

$$\Lambda_{24}=\begin{bmatrix} Z & Z & Z & Z & Z & Z & -i & Z \\ Z & Z & Z & Z & Z & Z & Z & -i \\ Z & Z & Z & Z & Z & i & Z & Z \\ Z & Z & Z & Z & Z & Z & i & Z \\ Z & Z & -i & Z & Z & Z & Z & Z \\ Z & Z & Z & -i & Z & Z & Z & Z \\ i & Z & Z & Z & Z & Z & Z & Z \\ Z & i & Z & Z & Z & Z & Z & Z \end{bmatrix}$$

$$\Lambda_{34}=\begin{bmatrix} Z & Z & Z & Z & -i & Z & Z & Z \\ Z & Z & Z & Z & Z & -i & Z & Z \\ Z & Z & Z & Z & Z & Z & -i & Z \\ Z & Z & Z & Z & Z & Z & Z & -i \\ i & Z & Z & Z & Z & Z & Z & Z \\ Z & i & Z & Z & Z & Z & Z & Z \\ Z & Z & i & Z & Z & Z & Z & Z \\ Z & Z & Z & i & Z & Z & Z & Z \end{bmatrix}$$

Let us define four-velocity $u^{\mu}=(u0,u1,u2,u3)=(\frac{1}{\gamma};\frac{\mathbf{v}}{\lambda})$. The matrices in the main equation (1.4.30) will be defined as:

$$\lambda^0 = 0 + \lambda^{01}u1 + \lambda^{02}u2 + \lambda^{03}u3 + \lambda^{04}$$

$$\lambda^1 = \lambda^{01}u0 + 0 + \lambda^{12}u2 + \lambda^{13}u3 + \lambda^{14}$$

$$\lambda^2 = \lambda^{02}u0 + \lambda^{12}u1 + 0 + \lambda^{23}u3 + \lambda^{24}$$

$$\lambda^3 = \lambda^{03}u0 + \lambda^{13}u1 + \lambda^{23}u2 + 0 + \lambda_{34}$$

The equation then will look as follows:

$$i\left(\lambda^0\frac{\partial\boldsymbol{\Phi}}{\partial x^0} + \lambda^1\frac{\partial\boldsymbol{\Phi}}{\partial x^1} + \lambda^2\frac{\partial\boldsymbol{\Phi}}{\partial x^2} + \lambda^3\frac{\partial\boldsymbol{\Phi}}{\partial x^3}\right) - m\boldsymbol{\Phi} = 0 \qquad (1.4.31)$$

The mass term of this equation will then be defined by the following correlation:

$$m = \int_V \left(\boldsymbol{\Phi}^+ \boldsymbol{\Lambda}_2 u^\mu \frac{\partial\boldsymbol{\Phi}}{\partial x^\mu} - u^\mu \frac{\partial\boldsymbol{\Phi}^+}{\partial x^\mu}\boldsymbol{\Lambda}_2\boldsymbol{\Phi}\right)\frac{dV}{\gamma}$$

because $\bar{\boldsymbol{\Phi}} = \boldsymbol{\Phi}^+ \Lambda$; $\Lambda_1 = \lambda^{04}\lambda^{14}\lambda^{24}\lambda^{34}$; $\Lambda = \lambda^{12}\lambda^{13}\lambda^{14}\lambda^{23}\lambda^{24}\lambda^{34}$; $\Lambda_2 = \Lambda\Lambda_1$

The explicit form of 4 matrices λ^μ depends on velocity, as well as of numerical matrices $\Lambda, \Lambda_1, \Lambda_2$ of the size 32x32. Using a good personal computer it is possible to prove the correctness of the correlations in (1.4.5) by making direct computations of the commutators and anticommutators with the help of symbol mathematics programs (Maple -18, Mathematica- 9).

4. Solve equations and deriving the value of the fine structure constant

There is a most profound and beautiful question associated with the observed coupling constant, e – the amplitude for a real electron to emit or absorb a real photon. It is a simple number that has been experimentally determined to be close to 0.08542455. (My physicist friends won't recognize this number, because they like to remember it as the inverse of its square: about 137.03597 with about an uncertainty of about 2 in the last decimal place. It has been a mystery ever since it was discovered more than fifty years ago, and all good theoretical physicists put this number up on their wall and worry about it.) Immediately you would like to know where this number for a coupling comes from: is it related to pi or perhaps to the base of natural logarithms? Nobody knows. It's one of the greatest damn mysteries of physics: a magic number that comes to us with no understanding by man. You might say the "hand of God" wrote that number, and "we don't know how He pushed his pencil." We know what kind of a dance to do experimentally to measure this number very accurately, but we don't know what kind of dance to do on the computer to make this number come out, without putting it in secretly! Richard P. Feynman (1985). "QED: The Strange Theory of Light and Matter", p. 129.

The attempts to solve equation of the (1.4.30), (1.4.31) type gave no result. However, [7, 8, 182, 196, 200, 201] an interesting was found for a modified scalar version of the integro-differential equation (1.4.30), which may be written down as follows:

$$\begin{gathered}\left(\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\frac{\partial}{\partial y}+\frac{\partial}{\partial z}\right)\Phi\left(x,y,z,t\right)\\ =-2i\Phi\left(x,y,z,t\right)\int\limits_0^x\int\limits_0^y\int\limits_0^z\Phi^*\left(x,y,z,t\right)\frac{\partial\Phi\left(x,y,z,t\right)}{\partial t}dxdydz\end{gathered}\qquad(1.4.32)$$

We will seek the solution of this equation in the form

$$\Phi(x,y,z,t)=F(x,y,z)exp(-i(\omega t-kx-ky-kz)),$$

where

$$F(x,y,z) = X(x)Y(y)Z(z)$$

and ω, k are some constant parameters. Substituting these expressions in (1.4.32), we obtain under condition $\omega = 3k$ following equation w. r. t X, Y, Z:

$$\frac{X'(x)}{X(x)} + \frac{Y'(y)}{Y(y)} + \frac{Z'(z)}{Z(z)} = -2\omega \int_0^x X^2(x)dx \cdot \int_0^y Y^2(y)dy \cdot \int_0^z Z^2(z)dz$$

Differentiating the left-hand and right-hand sides w. r. t. x, y, z successively, we obtain three equations for $X(x), Y(y), Z(z)$:

$$\begin{aligned}
\left(\frac{X'(x)}{X(x)}\right)' &= -2\omega X^2(x) \int_0^y Y^2(y)dy \cdot \int_0^z Z^2(z)dz \\
\left(\frac{Y'(y)}{Y(y)}\right)' &= -2\omega Y^2(y) \int_0^x X^2(x)dx \cdot \int_0^z Z^2(z)dz, \\
\left(\frac{Z'(z)}{Z(z)}\right)' &= -2\omega Z^2(z) \int_0^x X^2(x)dx \cdot \int_0^y Y^2(y)dy.
\end{aligned} \quad , \qquad (1.4.33)$$

Putting

$$U(x) = \int_0^x X^2(x)dx, \quad V(y) = \int_0^y Y^2(y)dy, \quad W(z) = \int_0^z Z^2(z)dz,$$

we obtain the system of ordinary differential equations for $X(x), Y(y), \ldots W(z)$:

$$X''-\frac{(X')^2}{X}=-2\omega X^3VW, U'(x)=X^2(x),$$
$$Y''-\frac{(Y')^2}{Y}=-2\omega Y^3UW, V'(y)=Y^2(y), \quad (1.4.34)$$
$$Z''-\frac{(Z')^2}{Z(z)}=-2\omega Z^3UV, W'(z)=Z^2(z).$$

Further, we have put the numerical value of ω, namely, $\omega=\frac{1}{2}$ and integrated numerically (with the help of Maple-18) this system under following initial conditions (reasonable from physical point of view):

$$X(0)=Y(0)=Z(0)=1, X'(0)=Y'(0)=Z'(0)=U(0)=V(0)=W(0)=0.$$

According to obtained solution X(x), Y(y), Z(z) are identical rapidly decreasing functions of following type:

$$X(x)\propto exp(-x^p),\quad Y(y)\propto exp(-y^p),\quad Z(z)\propto exp(-z^p),\quad 1<p<2. \quad (1.4.35)$$

The plot of X(x) is shown in Fig. 1.4.1. The basic equation (1.4.32) can be reduced to the scalar equation [6, 7, 8, 200, 201] for the density of the space charge of the space charge of the bunch, which represents the particles:

$$\frac{1}{c}\frac{\partial \Phi(r,t)}{\partial t}+\frac{\partial \Phi(r,t)}{\partial r}+\frac{4\pi i\Phi(r,t)}{\hbar}\int_0^r\left\{\Phi^*(s,t)\frac{\partial \Phi(s,t)}{\partial t}-\frac{\partial \Phi^*(s,t)}{\partial t}\Phi(s,t)\right\}s^2ds=0 \quad (1.4.36)$$

Let us solve this equation together with the Poisson equation [6, 7, 8]

$$\text{divgrad}\,\varphi=-4\pi\rho$$

We seek the solution in the form

$$\Phi(r,t)=\bar{F}(r)\exp[-i(\omega t-kr)] \quad (1.4.37)$$

We get the following system of equations if the condition

$$\omega = kc$$

is fulfilled:

$$\frac{d\bar{F}(r)}{dr} + \frac{8\pi\omega\bar{F}(r)}{h}\int_0^r s^2\bar{F}^2(s)ds = 0,$$

$$\frac{d^2\bar{\phi}(r)}{dr^2} + \frac{2}{r}\frac{d\bar{\phi}(r)}{dr} = -4\pi\rho(r) = -\frac{1}{2}\sqrt{\frac{c^3}{h}}\bar{F}^2(r), \qquad (1.4.38)$$

where

$$\rho(r) = \frac{1}{8\pi}\sqrt{\frac{c^3}{h}}\bar{F}^2(r)$$

is the electrical charge density. Let us suppose

$$x = \frac{r}{R}, \quad f(x) = \frac{\bar{F}(r)}{\bar{F}(0)}, \quad \bar{F}(0) \neq \infty$$

$$\rho(x) = \frac{2}{R^2\bar{F}(0)}\sqrt{\frac{h}{c^3}}\bar{\phi}(r)$$

$$K = \frac{8\pi\omega R^4\bar{F}^2(0)}{h}$$

System (1.4.38) can be expressed in dimensionless form:

$$\frac{d^2 \ln f(x)}{dx^2} + Kx^2 f^2(x) = 0 \qquad (1.4.39)$$

$$\frac{d^2\rho(x)}{dx^2}+\frac{2}{x}\frac{d\rho(x)}{dx}=-f^2(x)$$

As long as potential ρ with the accuracy up to an additive constant and its value does not affect the intensity of electrical field $\boldsymbol{E}=-grad\phi$, let us choose $\phi=0$. Due to the spherical symmetry in the centre of the particle, the condition $\boldsymbol{E}=0$ is fulfilled. Solving numerically the Cauchy problem for the system (1.4.39), taking the value $K=16\pi$ or (2x2x4π) and the initial conditions

$$f(0)=1,\quad f'(0)=0,\quad \phi(0)=0,\quad \phi'(0)=0 \tag{1.4.40}$$

we obtain the following integrals

$$I_Q=\int_0^\infty x^2 f^2(x)dx=8.5137256105758897351\cdot 10^{-2};\ I_Q^{\ 2}=1/137.9623876 \tag{1.4.41}$$

$$I_E=\frac{1}{2}\int_0^\infty x^2E^2(x)dx=5.6857305\cdot 10^{-3} \tag{1.4.42}$$

$$I_\mu=\int_0^\infty x^4 f^2(x)dx=3.2493214\cdot 10^{-2} \tag{1.4.43}$$

The quantity I_Q is a dimensionless electrical charge, which is brought to the following dimensional form of electrical charge Q:

$$Q=\sqrt{\hbar c}I_Q=4.78709\cdot 10^{-2}CGSE$$

This value is less than the modern experimental value of the electron's charge by only 0.3%. This is a fairly accurate number for the first theoretical attempt of the charge calculation. The plot of f(x) is shown in Fig. 1.4.1.

Thus it is not unusual to bring out the “corrections” of the J. Schwinger type to the integral (1.4.41)

$$I_e = I_Q + \frac{I_Q^2}{8\pi} - \frac{I_Q^3}{64\pi^2} = 8.5424692 \cdot 10^{-2},$$

which corresponds to the value of charge e = 4.803 2514 • 10^{-10} CGSE and the value of fine-structure constant $\alpha = 1/137.03552$.

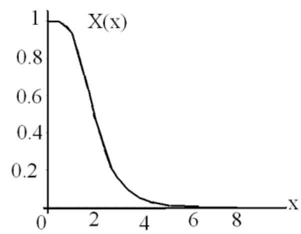

Fig. 1.4.1 *Density of the charge as function of the radius.*

This results is very important. There are some opinions: *"The mystery about α is actually a double mystery. The first mystery – the origin of its numerical value α ≈ 1/137 has been recognized and discussed for decades. The second mystery – the range of its domain – is generally unrecognized."* — Malcolm H. Mac Gregor, M. H. MacGregor (2007). The Power of Alpha. World Scientific.

"If alpha were bigger than it really is, we should not be able to distinguish matter from ether and our task to disentangle the natural laws would be hopelessly difficult. The fact however that alpha has just its value 1/137 is certainly no chance but itself a law of nature. It is clear that the explanation of this number must be the central problem of natural philosophy". — Max Born, A. I. Miller (2009). Deciphering the Cosmic Number: The Strange Friendship of Wolfgang Pauli and Carl Jung. W. W. Norton & Co.

Calculation spectrum masses all elementary particles see section 1.8.

The quantization of the electrical charge and masses seems to be the consequence of the balance between the dispersion and nonlinearity, which determines stable solutions.

The found density distribution for the particle's electrical charge allows us to determine the electrical form factor for the same particle

$$F(q) = \int_V \rho(x) \exp[-iqx] dV \tag{1.4.44}$$

We regret that we have not succeeded in finding an analytical solution of Eq. (1.4.39), but we are able to give a decent approximation. Let us look for a solution of Eq. (1.4.39) in the form

$$f(x) = \operatorname{sech} R(x) \tag{1.4.45}$$

Substituting Eq. (1.4.45) into Eq. (1.4.39) and taking into account that for small R we have

$$\frac{1}{2}\sinh 2R \approx R$$

we obtain

$$\left(RR^{'}\right)^{'} = 16\pi x^2\,;\quad R = \sqrt{\frac{8\pi}{3}}x^2 \tag{1.4.46}$$

$$f(x) = \operatorname{sech}\sqrt{\frac{8\pi}{3}}x^2 \tag{1.4.47}$$

It is interesting to note that if the particle’s 4-velocity is assumed to be zero at matrix Λ, then system (1.4.30) will reduce to eight similar Dirac equations.

5. Problems

In our view, although the Dirac equation describes the hydrogen atom spectrum absolutely correctly, it is not properly a fundamental equation. It has two weak points: the correct magnitude of the velocity operator's proper value is absent. It is known that in any problem of this type the proper value of the velocity operator is always equal to the velocity of light! In fact, Russian physicist and mathematician V. A. Fok regarded this as an essential defect of the Dirac theory.

The equations of the Unitary Quantum Theory we are proposing are more correct and fundamental. For this reason, a transition from correct fundamental equations to the incompletely accurate Dirac equation needs such a strange requirement as

$$u_\mu = 0$$

However, this requirement is absolutely unsatisfactory both from the physical and the mathematical points of view. Four-velocity has 4 components, of which three are usual components of the particle velocity along three axes, and they really can tend to zero. But the same cannot be done with the fourth component.

In the second paragraph of the preface of the book A History of the Theories of Aether and Electricity, by Sir Edmund T. Whittaker (Edinburgh, Scotland, April, 1951) was written the following: *"A word might be said about the title 'Aether and Electricity'. As everyone knows, the aether played a great part in the physics of the nineteenth century; but in the first decade of the twentieth, chiefly as a result of the failure of attempts to observe the Earth's motion relative to the aether, and the acceptance of the principle that such attempts must always fail, the word 'aether' fell out of favour, and it became customary to refer to the interplanetary spaces as 'vacuous'; the vacuum being conceived as mere emptiness, having no properties except that of propagating electromagnetic waves. But with the development of quantum electrodynamics, the vacuum has come to be regarded as the seat of 'zero-point' oscillations of the electromagnetic field, of the* *'zero-point' fluctuations of electric charge and current, and of a 'polarization' corresponding to a dielectric constant different from unity. It seems absurd to retain the name 'vacuum' for an entity so rich in physical properties, and the historical word 'aether' may be fitly retained."* Of course, now aether is not old aether of the nineteenth century, maybe it is Higgs field?

The question is that the main relativistic relation between energy, impulse, and mass

$$E^2 = P^2 + m^2 \tag{1.4.48}$$

has been still beyond any doubt. In particular, all of the previous equations are based on relativistic invariance. Nevertheless, we shall ask ourselves once again about what is happening with that relation at the exact moment when the wave packet disappears being spread over the space. At that moment the particle does not exist as a local formation. This means that in the local sense there is no mass, local impulse, or energy. The particle in that case, within sufficiently small period of time, is essentially non-existent, for it does not interact with anything. Perhaps this is why the relation (1.4.48) is average and its use at the wavelength level is equal or less than the De Broglie wavelength, which is just illegal. The direct experimental check of that relation at small distances and short intervals is hardly possible today. If the relation (1.4.48) is declined, then it may result in an additional conservation of energy and impulse refusal; but, as we know, according to the Standard Quantum Theory, that relation may be broken within the limits of uncertainty relation.

On the other hand, the Lorenz's transformations appeared when the transformation properties of Maxwell's equations were analyzing. However electromagnetic waves derived from solutions of Maxwell's equations move all in vacuum with the same velocity, i.e. are not subjected to dispersion and do not possess relativistic invariance. Our partial waves, which form wave packet identified with a particle, possess always the linear dispersion. Under such circumstances, it would be quite freely for authors to spread the requirement of relativistic invariance to partial waves. Such requirement has sense in respect only to wave packet's envelope, which appears if we observe a moving wave packet and his disappearance and reappearance. May be the origin of relativistic invariance would be connected in future with the fact that an envelope remains fixed in all inertial reference frames; only the wave's length is changed.

It's quite complicated [174-178, 186, 187]. The special relativity – is in fact Lorentz transformations (1904) derived by V. Vogt (1887) in the century before last. These transformations followed from the properties of Maxwell equations which are also proposed in the nineteenth century (1873). One of these equations connecting electrostatic field divergence and electric charge (Gauss' law of flux), in fact is just another mathematical notation of Coulomb's law for point charges.

But today anybody knows that Coulomb's law is valid for fixed point charges only. If charges are frequently moving Coulomb's law is not performed. Besides everybody knows that lasers beams are scattered in vacuum one over another, absolutely impossible in Maxwell equations. That means that Maxwell equations are approximate - and for the moving point charges experimental results essentially differs from the estimated ones in the case charges areas are overlapping.

Few people think about the shocking nonsense of presenting in any course of physics of point charge electric field in the form of a certain "sun" with field lines symmetrically coming from the point. But electric field – is a vector, and what for is it directed? The total sum of such vectors is null, isn't it?

There are no attempts to talk about, but such idealization is not correct. We should note that Sir Isaak Newton did not used term of a point charge at all, but it's ridiculous to think that such simple idea had not come to him! As for Einstein, he considered *"electron is a stranger in electrodynamics"*. Maxwell equations are not ultimate truth and so we should forget, disavow the common statement about relativist invariance requirement being obligatory "permission" for any future theory.

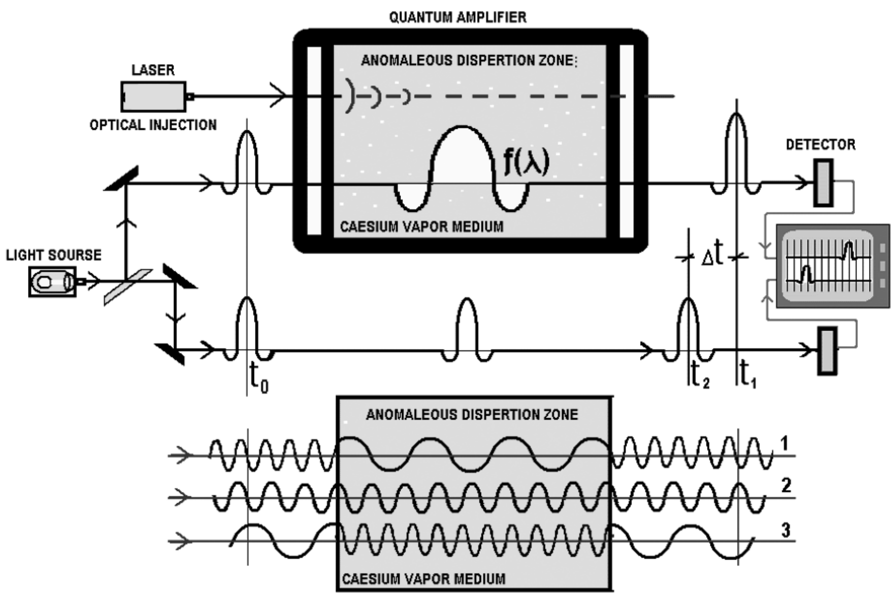

Fig. 1.4.2 *Wang experiment [169].*

To reassure severe critics we should note that UQT is relativistically invariant, it allows to obtain correct correlation between an energy and impulse, mass increases with a rate, while relativistic invariance just follow of the fact that the envelope of moving packet is quiet in any (including non-inertial) reference systems. To be honest we should note that subwaves the particles consist of are relativistically abnormal, at the same time envelope wave function following from their movement confirms terms of Lorentz transformations.

The success of Maxwell equations in description of the prior-quantum view of world was very impressing. Its correlation of the classical mechanics in forms of requirement to correspond Lorentz transformations was perfectly confirmed by the experiments that all these had resulted in unreasoned statement of Maxwell equations being an ultimate truth…

Other reasons for this were later very carefully investigated by a disciple of one of the authors (L. S.), Professor Ratis Yu. L. (S. Korolev Samara State Aero-Space University), who has formulated the modern spinor quantum electrodynamics from the UQT point of view:

1. Maxwell equations contain constant c, which is interpreted as phase velocity of a plane electromagnetic wave in the vacuum.

2. Michelson and Morley have never measured the dependence of the velocity of a plane electromagnetic wave in the vacuum on the reference system velocity as soon plane waves were mathematical abstraction and it was impossible to analyze their properties in the laboratory experiment in principle.

3. Electromagnetic waves cannot exist in vacuum by definition. A spatial domain where an electromagnetic wave is spreading – is no longer a vacuum. Once electromagnetic field arises in some spatial region at the same moment such domain acquires new characteristic – it became a material media. And such media possesses special material attributes including power and impulse.

4. Since electromagnetic wave while coming through the abstract vacuum (the mathematical vacuum) transforms it in a material media (physical vacuum) it interacts with this media.

5. The result of the electromagnetic wave and physical vacuum interaction are

compact wave packets, called photons.

6. The group velocity of the wave packet (photon) spreading in the media with the normal dispersion is always less its phase velocity.

The abovementioned allows to make unambiguous conclusion:

the main difficulties of the modern relativistic quantum theory of the field arise from deeply fallacious presuppositions in its base. The reason for this tragic global error was a tripe substitution of ideas – velocity of electromagnetic wave packets 'c' being transformed in numerous experiments physics have construed as constant 'c' appearing in Maxwell equations and Lorentz transformations. Such blind admiration of Maxwell and Einstein geniuses (authors in no case do not doubt in the genius of these persons) had led XX century physics up a blind alley. The way out was in the necessity of revision of the entire fundamental postulates underlying the modern physics. Exactly that was done by UUQFT [165, 166].

Some time ago CERN has conducted repeated experiments of the neutrino velocity measurement that appeared to be higher than velocity of the light. For UUQFT they were like a balm into the wounds. In fact the movements in excess of the light velocity were discovered earlier by numerous groups of researches. The most interesting were experiments of [169] (Wang, 2000, Princeton, USA), they had disclosed velocities 310 times higher than the light.

Nearly everybody disbelieved it. And now the neutrino movements exceeding the velocity of the light were disclosed in CERN. The importance of these experiments for UUQFT is settled in the article [166] where at the page 69 it is written that "this should be considered as direct experimental proof of UUQFT principle".

There are also other ideas [190, 191]. For example, at «New Relativistic Paradoxes and Open Questions», by Florentin Smarandache, shows several paradoxes, inconsistencies, contradictions, and anomalies in the Theory of Relativity. According to the author, not all physical laws are the same in all inertial reference frames, and he gives several counter-examples. He also supports superluminal speeds, and he considers that the speed of light in vacuum is variable depending on the moving reference frame.

The author explains that the red shift and blue shift are not entirely due to the Doppler Effect, but also to the medium composition (i.e. its physical elements, fields, density, heterogeneity, properties, etc.). Professor Smarandache considers that the space is not curved and the light near massive cosmic bodies bends not because of the gravity only as the General Theory of Relativity asserts (Gravitational Lensing), but because of the Medium Lensing.

In order to make the distinction between "clock" and "time", he suggests a first experiment with a different clock type for the GPS clocks, for proving that the resulted dilation and contraction factors are different from those obtained with the cesium atomic clock; and a second experiment with different medium compositions for proving that different degrees of red shifts/blu shifts would result. To regret, the authors today have no decisive position to these complicate questions.

Note, this question is terribly complicate and probably is to be leaved to next generations. From one side, the time in UQT exists, so to say, in our head only. From other side, the Lorenz Transformations describe correctly some experimental facts, for example, the mass growing with velocity. Otherwise, all atomic accelerators would be out of order. Thereafter, it is a big mistake to consider all Special Relativity Theory as erroneous. The attitude to the Special Relativity Theory is today highly vague and may be compared in full with the discussion among painters about significance of the Malevitch picture "The black square".

It is curios but from another side the Special Relativity Theory declares that the spreading velocity of the information and of the signals cannot exceed the light velocity. At the same time today it is well known that the gravity interaction spreads with the velocity exceeding many times the light velocity. Laplace has obtained corresponding estimates long ago. But this problem is not discussed somehow in Special Relativity.

As soon relativistic invariance underlies every of the numerous quantum theories of the field, it leaves a devilish imprint at everything. Nevertheless relativistic ratio between energy and impulse although being absolutely correct in fact are not obligatory follow from relativistic invariance only and can result from another mathematical reasons that will be discovered in future. Nowadays Standard Model (SM) combines the most elegant mathematical miracles of researches which hands were tied with relativistic strait-jacket and it not so bad describes these experimental data. Amazing that it was possible to think it out at all.

1.5 Interpretation of Unitary Quantum Theory

"…There is not now, has not been, nor will there be from now on knowledge more certain to affect you than that I'm going to give you, because it will send you out of your mind - so strikingly simple, bright, and immense is it."

- from Oriental folklore

1.5.1 Non-relativistic Case

The envelope of the wave function $\Phi(x,t)$ describes a wave packet's field transformation within its motion. There are points at which the packet/particle disappears, $\Phi(x,t) = 0$, yet particle energy remains in the form of harmonic components that produce field vacuum fluctuations at some point in space-time. Neither the value nor moment of these fluctuations' appearance nor the background flux at that point depends on the apparent distance to such a vanished particle. This precept does not violate the principles of relativity, however, in that the apparent background does not transfer any information.

Our real 'world' continuum consists of enormous quantity of particles moving with different velocities. Partial waves of the postulated vanishing particles create real vacuum fluctuations that change in a very random way. Certain particles randomly appear in such a system, owing to the harmonic component energy of other vanished particles. The number of such "dependant particles" changes, though; they suddenly appear and vanish forever, as the probability of their reappearance is negligibly small, and so we do not expect that all particles are indebted to each other for their existence.

Yet, if some particles are disappearing within an object, other particles are arising at the same moment in that object due to the contribution of those vanishing particles' harmonic components – and vice versa. The simultaneous presence of all of the particles within one discrete macroscopic object is unreal. Some constituent particles vanish within the object while others appear. In general, a mass object is extant overall, but is not instantaneously substantive and merely a 'false' image. It is clear that the number of particles according to such a theory is inconstant and all their ongoing processes are random, and their probability analysis will remain always on the agenda of future research.

In reality, the hypothetical measurements considered before (in section 1.2) are impossible, because all measuring instruments are macroscopic. Since the sensor of any such device is an unstable-threshold macro-system, only macroscopic events will be detected, such as fog drops in a Wilson chamber, blackening of photo-emulsion film, photo-effects, and the formation of ions in a Geiger counter. Within macro-devices of any type, the sensor's atomic nuclei and electron shells are in close proximity, creating a stable system which is far from being able to take on all arbitrary energy configurations that might be imagined.

The nature of that stable condition is allowed for a series of numerous but always-discrete states only, and the transition from one state to another is a quantum jump. This is why absorption and radiation of energy in atomic systems takes place by quanta, and is a consequence of subatomic structure. In other words, quantization appears because of bound states arising, with 'substance' being the richest collection of an enormous number of bound states. However, it is known that free particles may vary their energy continuously.

However, this does not mean that while passing from one quantum-mechanical system to another, the quantum or particle remains as something invariable and indivisible. Particle energy can be split up and changed due to vacuum and external field fluctuations, but the measuring conditions of our devices are such that we are able to detect quite definite and discrete particles only.

The wave packet/particle exhibits periodicity following our UQT approach, and the mass of a moving particle such as a proton changes from its maximal value to zero and back again – running the series of intermediate values corresponding to the masses of mesons. For example, it might be said that the proton takes, during some intervals of time, the form of a π-meson. This phenomenon is confirmed by numerous experiments, which are explained in classical quantum theory in another way: the proton is permanently surrounded by a cloud of π-mesons, an explanation which is in essence equivalent to our model.

Any 'normal' measurement, in the long run, is based on the interchange of energy and is an irreversible process. That is why the particle interferes in the state of macro-device giving up (or acquiring in the case of devices with inversion) quantum of energy Θ. The best measuring instrument will be one wherein the discrete threshold energy Θ which characterizes device instability is absolutely minimal. With a hypothetical measurement $\Theta = 0$, such that the researcher does not influence the particle with his sensor, then such a device would have 100% effectiveness and could detect vacuum fluctuations.

The measuring instrument should be so that eventually only its classical characteristics are used for its work; in other words, Planck's constant should not play any role in it after the initiation. Such a device is as much as possible (but not totally) free from statistical effects. Thus in measuring processes particle detectors are those reference frames in what respect according to the quantum theory the system's state is to be determined.

Let us consider the process of particle – macro-device interaction. Particle energy periodically changes with frequency ω_B and vacuum fluctuations (additionally changing the energy) are imposed at it in a random way. To detect the particle, the macro-device has to wait until particle total energy $|\Phi|^2$ and vacuum fluctuations ε exceed the operation threshold Θ of the device:

$$\varepsilon + |\Phi|^2 \geq \Theta \tag{1.5.1}$$

The energy of vacuum fluctuation ε depends on the total number of the particles in the Universe and is created thanks to the particles disappeared. As far the contribution of each partial wave in every point is infinitesimal (its distribution law may be any) in accordance with central limit theorem of Alexander Lyapunov the summary background to be formed by tremendous number of particles and their partial waves will have a normal distribution with maximal entropy. The probability P of vacuum fluctuations with the energy more than ε_0 is equal to

$$P = \frac{1}{\sqrt{(2\pi)}\sigma} \int_{\varepsilon_0}^{+\infty} \exp\left(-\frac{\varepsilon^2}{2\sigma^2}\right) d\varepsilon \qquad (1.5.2)$$

and the value σ (dispersion), depending on the particles' number within the Universe is considered in our case as constant. *The theory under consideration requires finiteness of* σ *, and then finiteness of the Universe.* By using (1.5.2) and the Moivre-Laplace formula, we obtain for the probability of the particle detecting the following expression [4, 5]:

$$P = \frac{1}{2} erfc\left(\frac{\Theta - |\Phi|^2}{\sigma^2}\right) \qquad (1.5.3)$$

It is evident from the last formula that the probability of the particle's detecting depends on the sensitivity of the measuring instrument. A more rigorous approach to the theory of quantum measurements will be considered in the next sect.1.6.

The developing point of view results in the conclusion that relation $E = \hbar\omega$ is fulfilled at the atomic level only. Thus the particles may exist (after fragmentation on the mirror) with similar frequency ω_B , but with different wave amplitudes f, and so with different probabilities to be detected. One of the particles being split up at the mirror or grid may be detected in a few points at once. The other particle may disappear completely, making its contribution in vacuum fluctuations without any marks.

Following P. Dirac, the photon may interfere only on its own and so the translucent mirror splits it into two parts. According to standard quantum theory, the photon is not able to split with frequency conservation, so it is assumed that two separate photons may interfere at terms they belong to one mode, which occurs in the case of the translucent mirror. However, according to UQT, photons are constantly splitting at the translucent mirror with frequency conservation, but the probability to detect such splitting photons is reduced.

An uncertainty relation results from the fact that energy and impulse are not fixed values, but periodically change due to the appearance and disappearance of the particle. That question is examined in detail in sect. 2.13. Due to the statistical measuring laws, it is impossible to measure energy and impulse by macro-devices especially because of principal and not-foreseen vacuum fluctuations. On the other hand, for the hypothetical researcher the centre of the wave packet has exact coordinates, impulse, and energy at the given moment of time. However, neither we nor the hypothetical observers are able to predict exactly its value at the following moment. Moreover, we (macro-researchers) do not have even a method of accurate measuring, for the process of macro-devices measuring is statistical.

The presence of vacuum fluctuations makes microcosm laws for each researcher statistical in principle. The exact prediction of the events requires the knowledge of the vacuum fluctuation's exact value in any point and at any moment of time. This is impossible, because it requires the information about behaviour and structure of all various wave packets within the Universe and also the possibility to control their motion.

Werner Heisenberg wrote [27]: *"If we would like to know the reason why α -particles are emitting at an exact moment we must, apparently, know all microscopic states of the whole world we also belong to, and that is, obviously, impossible."*

This is why the conclusion that Laplace determinism is lost within the modern and future physics of microcosm shall be considered ultimate. The same point of view about the reason of the arising of probability approach in quantum mechanics was expressed by R. Feynman [18]: *"There is almost no doubt that it (probability) results from the necessity to intensify the effect of single atomic events up to the level detectable with the help of big systems."*

It is good to remember the deep and remarkable words of J. Maxwell: *"The calculation of probabilities is just the true logic of our world."*

The most impressive demonstration of the random chaotic nature of all quantum processes can be seen at the start of a nuclear reactor. Chaos of micro-effects at a low level of average power results in enormously huge fluctuations of chain reactions, which exceed to a considerable extent the average level. Atomic chaos manifestations always exasperate the participants and sometimes create a threatening impression of the processes' uncontrollability with all following consequences. However, cadmium rod removal precipitates smoother fluctuations.

The envelope of partial waves appearing in the result of linear transformations of wave packet as well as in the result of it splitting and fragmentation satisfies the C. Huygens principle. This explains the way of possibility to connect the formally moving particle and plane monochromatic de Broglie wave as it spreads in the line of motion and all the wave properties of particles (such as interference and diffraction) also.

For example, let the wave packet run up to the system with two slots. Each of the wave packet of harmonic components interferes at these slots. There would be an interference pattern of each harmonic component at the screen (since harmonic components amplitudes are extremely small, it may be not possible to see it). However, above this interference pattern the other interference patterns of an infinite large number of the other harmonic components are superimposed. The general composition results in the long run interference pattern of the de Broglie wave envelope.

For the total reversibility of quantum processes, it is necessary while exchanging +t for -t not only to reproduce the amplitude and form of the packet at +t, but also to restore the background fluctuation. The equations of quantum mechanics permit formal exchange of +t for -t under the condition of simultaneous exchange Φ for Φ^*, i.e. formal reversibility (the amplitude and form of the packet reproduction). Actually, such reversibility does not exist in nature even for the hypothetical observer, as for reproduction of the former vacuum fluctuations the reversibility of all processes in the Universe is required, and that is impossible. *However, one can to think that in terms of Unitary Quantum Theory the reversibility has a statistic character (single processes may be reversible with define probability).*

Introduced function Φ has a strictly monochromatic character, but does not exist as a real plane running wave. Although this function corresponds to the particle's energy, other notions may also agree with it: "Waves of probability", "informational field", and "waves of knowledge". As stipulated by A. D. Alexandrov and V. A. Fok [28] a wave function has sense for a separate system, but we can pick it out only by numerous similar experiments and after averaging, though the hypothetical researcher is able to measure this wave function for one particle. It is interesting that the envelope remains fixed within all inertial coordinates systems (only the wave length is changed).

Function Φ may also be connected with wave function Ψ of quantum mechanics describing the plane wave moving in the space. However, the value Φ^2 differs from $\Psi\Psi^*$ not only by presence of frequent oscillations. With Φ^2 the particle's energy is connected, but with $\Psi\Psi^*$ only the probabilities connect.

In standard quantum theory it is not so easy. When comparing mathematical expressions for the density matrix in quantum mechanics and the correlation function of random classical wave field, we find them quite similar, although they describe absolutely different physical objects. In the simplest cases the wave function relates to a single particle and has any sense in the presence of the particle only. Wave function has no sense in those areas where particle is absent. More formally, according to quantum theory, physical values can be obtained in the result of either one or other operators' acts on wave function. Then the average values may be computed by averaging with some weight. That is why notions of absolute phases and amplitudes have no physical sense and may be selected arbitrary for usability only. Large relative changes of the amplitude in far situated points do not result in physical values changes if the wave function gradient is being transformed slightly. So $|\Psi|^2$ have a probability distribution sense but not the sense of real wave motion density as it were in the case of classic fields.

In contrast to ordinary quantum theory the phase plays quite essential role according to our approach. For example, if a particle reaches the potential barrier being in phase of completely vanishing ($\Phi(x,t)$=0), then due to linear character and superposition at small $|\Phi|$ it penetrates the quite narrow barrier without any interactions (Fig. 1.5.1).

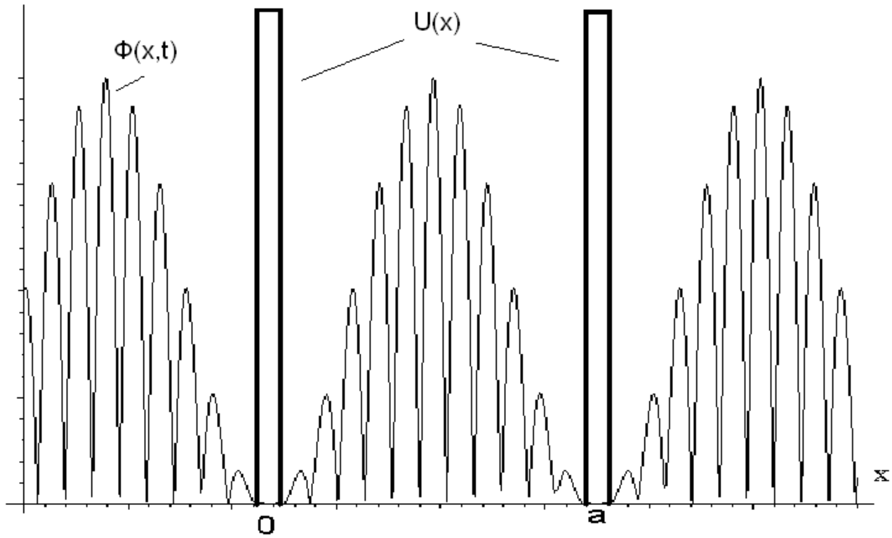

Fig. 1.5.1 *Particle penetrates the quite narrow barriers without any interactions.*

At the other hand, if the phase is so that value of $|\Phi(x,t)|$ is maximal, then due to non-linear character interactions would began and the particle might be reflected. That idea results in new effect: if there is a chain of periodical (with period a), narrow enough (in comparison with λ_B) potential barriers, bombarded with monochrome particles flux, then abnormal tunneling is to be considered at λ_B =2a, but that does not exist(?) in standard quantum theory [29].

Mathematically the process of the packet's appearing and vanishing without changing its character is possible as it is shown at Fig. 1.5.1. It enables formally to understand the fundamental fact of two different amplitude interference rules: for bosons when amplitudes interfere with equal signs and for fermions – with different signs (Fig. 1.2.1).

1.5.2 Relativistic Case

It may be all is vain,

Just un-experienced soul illusion ...

A. S. Pushkin

Analyzing (1.3.1) one can see that wave packet Φ contains oscillations term with frequency $\omega_s = \dfrac{mc^2}{\hbar\gamma}$ that corresponds to Schroedinger vibration. The physical meaning of that very quick oscillating process is as follows: after "Creator" having stirred up "the medium" created wave packet the last begins oscillating like membrane or string with frequency ω_s. Within the motion de Broglie vibrations is arising with frequency $\omega_B = \dfrac{mv^2}{\hbar\gamma}$ due to dispersion. At small energies $\omega_s >> \omega_B$ and in the presence of quick own oscillations have no influence on experiment and all quantum phenomena result from de Broglie oscillations. The value of frequency ω_B tends to ω_s with growth of energy and resonance phenomenon appears that result in oscillating amplitude increase in mass growth also (Fig. 1.5.2). Thus the well-known graph of particle mass dependence on the velocity approaching to light's velocity constitutes actually a half of usual resonance curve for forced oscillation of harmonic oscillator if energy dissipation is absent.

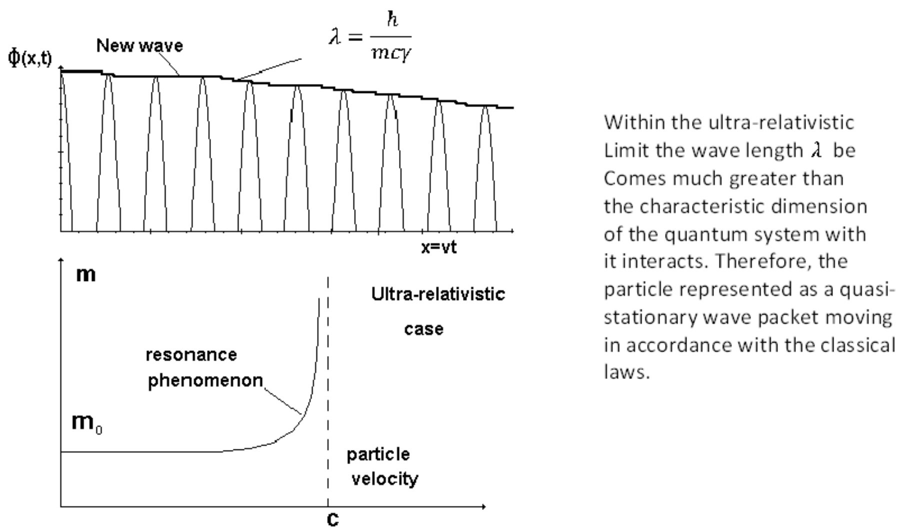

Fig. 1.5.2 *Appear of the New Wave in the ultra-relativistic limit.*

In the case when $v \to c$, frequency $\omega_B \to \omega_s\ \gamma \to 0$ beats appear with resonance frequency $\omega_d = \omega_s - \omega_B \approx \frac{mc^2\gamma}{\hbar}$, and particle will obtain absolutely new low-frequency envelop with wave length $\Lambda = \frac{h}{mc\gamma}$ (new wave). In ultra-relativistic limit case the value of Λ becomes much greater as typical dimension of quantum system it (new wave) interacts with. Now the length of new wave grows with energy contrary to de Broglie wave length slowly decreasing, and particle requires the form of quasi-stationary wave packet moving in accordance with classical laws. That explains the success of hydrodynamics fluid theory concerning with numerous particle birth when the packet having extremely big amplitude is able to split into series of packets with smaller amplitudes. But such splitting processes characterize not only high-energy particles. Something like this takes place at small energies also, but overwhelming majority of arising wave packets is under the barrier and so will not be detected. It would be perfect to examine by experiments at future accelerators the appearance of such new wave with the length growing together with energy.

But there is one more sufficiently regretting consideration. Due to our point of view relativistic invariance of equations should be apparently changed for something else. In fact the classical relativistic relation between energy and impulse

$$E^2 = P^2 + m^2$$

is doubly true for extra short intervals of time and small particle's displacement (equal to parts of de Broglie wave length). This relation is the result of averaging. What happens with particle impulse and mass when the packet is spread all over the Universe? Possibly they go to zero, but particle's energy as integral of all harmonic components squares sum remains constant (no wave dissipation) and the above-mentioned relation breaks. And probably the fundamental equation (1.4.30) should be written in any other form. But to be sure that equation should be solved first.

1.5.3 Possible Experimental Tests and Results

All ideas that have significance consequences are always simple

Leo Tolstoy

The developed theory will remain a freak of the imagination if following effects will not be experimentally confirmed:

1. Let very weak source emits by parallel bunch of N particles per 1 sec. If place in front of it gate will be opened during the experiment for short interval

$\tau << \frac{1}{N}$, then most probably that no one particle will penetrate, or they will be able to do it one by one. Let these particles fall down on the angle 45 degrees at translucent mirror (Fig. 2.13.1). According to ordinary quantum mechanics the particle will either penetrate the mirror or reflect. In accordance with the point of view described at that monograph the bunch will be split up at the mirror into two, three… of smaller bunches that depends on bunch phase in front of the mirror and on structure of mirror in given place. In general we will get two non-similar wave packets (under-thresholds particles or particles converted into state of phantoms) with smaller amplitudes. There is no change of frequency ω in formula $E = \hbar\omega$ (reddening), because all processes are linear, i.e. do not depend on amplitude. Besides the particle energy $|\Phi|^2$ is decreasing, that results in reducing of detection probability (for detection considerable vacuum fluctuation is necessary, but the probability of it appearance is too small). So, sometimes during process of measuring some particles should disappear or visa versa two particles should appear instead of one. The appearance of two particles from one does not contradict to energy conservation law, as far as the energy of under threshold particle may be increased up to the necessary level due to fluctuations.

For the first time such experiment over photons was carried out by Kozins [30]. He placed photo-multiplier tubes within each bunch and had detected few cases when a coincidence took place. He assumed these to be resulted from activity of independent photons being accidentally almost near and following each other in short time intervals. Unfortunately he did not carry out statistical verification of that assumption.

For the time being a spicy situation is arisen. A lot of experiments have been carried out similar to Kozins' one (for example first R. Hanbury Brown and

R. Q. Twiss experiments, J. F. Clauser [31-33]) resulted in conclusions that particles always had distinct tendency to reach detectors in correlated pairs (!). That result confirms the one we mentioned above. Amusingly, that some physicists had invented special devices of coherent state type for explanation of these experiments refuting standard quantum mechanics.

Late the experiments with delayed choice were carried out also confirming the developing in our book point of view. The description of these experiments can be found at "Scientific American" magazine under the title "Quantum philosophy". And quite recently the effect of electron division into two electrons (!) has been experimentally detected [34-35].

If those results were true, then it would be the most direct confirmation of UQT and total disaster for the ordinary quantum theory. Unfortunately till now nobody has taken into his head to interpret the results of all such experiments in this way, because energy conservation law formally prohibits it. The last is thoroughly checked at very high levels of energy, and since the energy in that case considerably exceeds the energy of vacuum fluctuation, everything is held true. But at small energies nobody studied that question directly. We should repeat once again that any result to be obtained at small energies for one definite particle is random; more over the indeterminateness principle gives no opportunity to detect something precisely for separate particles.

We should specially dwell on J. Bell inequalities (or theorem). There is a perfect review made by J. F. Clauser and A. Shimony [36] that most likely proves our point of view. Such approach is absent in the researches of many physicist and they, to make both ends meet, are obliged to assume over-light velocities [37] and even fantastic processes of "teleportation" [38] and "telepathy" (in connection with this see theorem of Kochen and Specker [39]).

2. Let us assume that monochrome particles with energy $E > \Theta$ participate diffraction experiments. If only one separate particle diffracts, then it creates some interference pattern, but it can not show itself in i maximums because packet energy within it $E_i < \Theta$. If at the same moment n coherent particles interfere, then the energy in the maximums can increase due to superposition of different particle's fields, and the device will be able in this case to detect it. Thus interference pattern of particles with small energies while transforming into a flux of separate particles should disappear. That effect was studied experimentally [40], till now it does not have any satisfactory explanation. In the case $\mathrm{E} >> \Theta$, as it were in experiments [41], that effect will not take place.

3. The coefficient of passing of any coherent particles with small energies ($\lambda_B \cong 0.5A$), through the series of periodical potential barriers (mono-crystal) will be maximal at ($\lambda_B = 2a$), where a is the target grid mono-crystal constant (Fig. 1.5.1). The same, but less weaker effect should becomes appeared again at ultra-relativistic energies, when $\Lambda = 2a$. To run such experiments the flux of mono-energetic and synchronous in phase particles is required. It can be obtained by selecting narrow packet of particles reflected from mono-crystal.

4. In connection to the fact that slowly changing part of space-time generates a field, and local hump of that field is a periodically disintegrating and appearing particle, the theory cannot consider processes not satisfying the field laws. Then un-removable vacuum fluctuations really existing will be in such theory non-invariant relative to rotations, transmissions, space and time reflections and so on [42], and, therefore, conservation laws concerned with them will be non-local and approximate. Such infringements easily arise when particle energy $|\Phi|^2$ is of the same range as dispersion σ of vacuum fluctuations.

Unfortunately, these processes will arise near the threshold and therefore they are difficult for investigation.

5. Since every particle with very small probability can spontaneously arise from vacuum or vanish, all chemical elements are subjected to absolutely new type of nuclear transformations: any element may be transformed into his isotope or into one of his nearest neighbour in periodic table. Upon a time (1905), E. Rutherford pointed it out [43], and these processes were really discovered in geology, but they do not have any explanations yet [44] (within standard quantum mechanics).

6. At collision of any particles of the processes of mutual penetration without any other interaction are to be detected in the case when in the point of collision one of particles or both will spread. It seems, s –state of hydrogen atom is a good illustration of that. We should note that the same phenomena have appeared in Bohr-Sommerfeld model (pendulum orbits) too, but were rejected at once by standard quantum theory as quite preposterous.

It is quite appropriate to quote one more statement of one of quantum theory founders (quite disavowing this theory, but almost unknown – why? – among broad scientific community):

"There are many experiments that we are just not able to explain if we don't consider the waves as namely waves exerting its influence upon all region, where they spread, and assume the location of these waves being "possibly here, possibly there according to probabilistic viewpoint". E. Schroedinger, Brit. J. Philos. Sci., vol. 3, page 233, section 11, 1952.

The offered picture of unitary quantum mechanics for a single particle from the position of united field is rather simple and obvious from hypothetical observer's point of view. If a hypothetical observer usually can measure the value of the wave function amplitude, we can not do it at all. We have to be satisfied with its probability interpretation keeping in mind that rather very simple mechanism is hidden behind and this mechanism open the way for explanation of quality transformations of quantum phenomena, and allows to reduce the description of the whole nature to description of some united field, and the continuous transformations of that field show the astonishing variety of phenomena being under observation.

In spite of mathematical complexity quantum theory will stop being paradoxical and frank words of Richard Feynman [45] *"I can easily say that nobody understands quantum mechanics"* will become the property of history.

In conclusion we would like to quote extremely acute words of Louis de Broglie: *"Those who say that new interpretation is not necessary I would like to note that new interpretation may have more deep roots and such theory in the long run will be able to explain wave-particle dualism, but that explanation will not be received either from abstract formalism, modern nowadays, or from vague notion of supplementary. But I think that the highest aim of the science is always to understand. The history of the science shows if any time somebody succeeded in deeper understanding of physical phenomena class, new phenomena and applications appeared. Hope that many researchers will study that enthralling question casting aside preconceived opinions and not overestimating the importance of mathematical formalism, whatever beautiful and essential it was, because that may result in loss of deep physical sense of phenomena"* (Louis de Broglie, Compt. Rend, 258, 6345, 1964)

1.6 The Theory of Optimal Detector and Quantum Measurements

The truth is too fine a matter and our instruments are rather blunt to touch the truth without any damage. While reaching the aim they crush it and move aside rather false than true.

Blaise Pascal.

The problem of the measuring device MD in any quantum theories may be reduced to the following formula [4, 5, 200, 201]:

$$MD=A+D$$

where A is an analyzer and D a detector. The analyzer performs the spectral decomposition into pure states of the measured dynamical variable L. This role may be played by the magnetic field, various diffractive systems, and polarisers. The theory of this device will not be considered here. The detector changes its state under the influence of the particle and this change is always a microscopic phenomenon. The role of detectors is played usually by highly complex macroscopic systems, such as the sensitive grain of photo emulsion, super cooled gas in cloud chamber, an electron avalanche in Geiger and so on.

Let us denote by Q the set of dynamical variables of the detector, with the help of which the states of the detector and the changes of these states are described. Since the detector is a macroscopic system, it is better to describe it not by means of wave functions $\Psi(Q)$, but by means of the density matrix $\rho_D(Q,Q')$. That is why the particle interacting with the detector is better described by the density matrix $\rho_m(x,x')$ also instead of the wave function $\Psi_m(x)$.

During the interaction between particle and detector, both density matrices should be combined into one common density matrix and this now depends on time:

$$\rho_{D+M} = \rho_{D+M}\left(Q, x, Q', x', t\right)$$

and satisfies the equation of motion

$$H = H_D(Q) + H_\mu(x) + W_{D\mu}(Q, x) \tag{1.6.1}$$

where $H_D(Q)$ is the Hamiltonian of the detector, $H_\mu(x)$ the Hamiltonian of the particle and $W_{D\mu}(Q,x)$ - the operator describing the interaction between the detector and the particle.

The common density matrix without any restrictions may be represented in the form

$$\rho_{D+M}\left(Q, x, Q'', x', t\right) = \sum \Psi_m^*(x)\, \rho_{mn}\left(Q, Q'\right) \Psi_n\left(x'\right),$$

where $\Psi_n\left(x'\right)$ are the eigenfunctions of the measured quantity - L. In general, this matrix is nondiagonal in respect to L. At work by $t \to \infty$ detector and its dynamical variables Q lie in some interval

$$Q_n' < Q < Q_n''$$

elements of the matrix are zero:

$$\rho_{mn}\left(Q, Q', t\right) = 0$$

besides

$$\rho_{mn}\left(Q,Q^{'},t\right)\neq 0$$

when

$$Q_n^{'}<Q\,,\;\; Q^{'}<Q_n^{''}$$

These equations are describing the interference of separate particular states of microsystem $\Psi_n(x)$ and their destruction by corresponding changed value of dynamic variable $L=L_n$ for Q state of detector. The stated measurement technique is primarily general in quantum mechanics and includes not only the microsystem itself but also both other parts of the device, the analyzer and the detector. However, all these equations can be easily written down. On the other hand, the solution is rather difficult.

Measurement problem with standard quantum theory is based on two different points of view:

The results of quantum effects' measuring are random and the theory deals with probabilities proportional to wave function amplitudes' squares. The amplitude will be depends on device and macro conditions exactly. This general point of view traces back to N. Bohr.

It is assumed that random measuring results conceal more complicated physical situation, and there are numerous variants of approaches using hidden parameters. Nowadays after the experimental verification of Bell's inequalities the first point of view is winning.

In the case of particle representation as a wave packet bunch and the detector as to some extended as a threshold device, one can evolve the approach proposed in [4-5] something further.

A general approach to the solution of this problem is described in 1.5. Now we proceed to its precise mathematical solution and to define the requirements that the macro instrument should meet if measurements are to be made with minimal errors. Note therewith that a similar need in identification of the useful signal from noise arose in radiolocation and was initially resolved in [46] by W. W. Peterson, T. G. Birdsall, and W. C. Fox.

Without entering into details of the interaction between quantum particles with macro instruments, which have been partially discussed in sections 1.2-1.5, the problem of particle recording or detection can be stated as follows:

On a wave packet with value $|\Phi|$ a vacuum fluctuation with value ε is additively imposed. For simplicity, let us regard the problem as single-dimensional and the eigenregion of the field as a segment of the numerical axis. Mark on that axis x a certain threshold value (Fig. 1.6.1)

$$\theta < a = |\Phi|$$

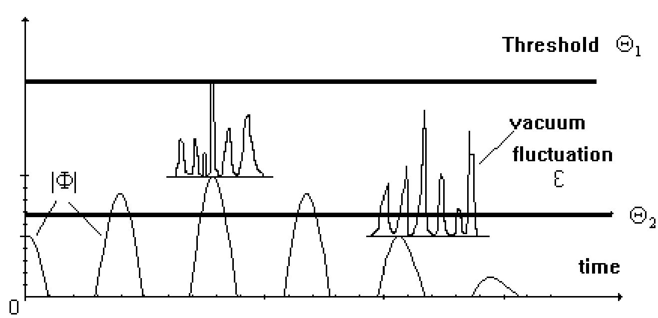

Fig. 1.6.2 *The distribution of the vacuum fluctuations.*

and let the eigenregion of the acting field be $\theta < x < \infty$. The measuring macro instrument distinguishes two situations. If there is a particle, then the value of the field which acts on the instrument is a + ε; if there is no particle, the value is ε. The instrument responds (the particle is recorded) when the value of the acting field exceeds a certain threshold θ, and then θ^2 is the minimal quantum energy for the macro-instrument to respond (sensitivity). Let us find the probability of error of the instrument. Let the distribution of vacuum fluctuations $W_a(x)$ be the distribution of the sum of the particle field and vacuum fluctuations $W_0(x)$ The conditional probability of failing to detect a particle when this goes through the macro instrument is (it is the case of $\theta = \theta_1$ in Fig. 1.6.1)

$$p_a(0) = p\{a + \varepsilon < \theta\} = \int_{-\infty}^{\theta} W_a(x)dx$$

and the conditional probability of detecting a particle when it is not there is

$$p_0(a) = p\{\varepsilon > \theta\} = \int_{\theta}^{\infty} W_0(x)dx$$

Let p(a) and p(0) be a priori the probabilities of particle flight or absence. Then the total probability of error is

$$p_{error} = p(a)\,p_a(0) + p(0)\,p_0(a) = p(a)\int_{-\infty}^{\theta} W_a(x)dx + p(0)\int_{\theta}^{\infty} W_0(x)dx$$

An instrument whose p_{error} is minimal can be viewed as optimal. When the threshold θ is lowered, the instrument sensitivity increases and thus the number of undetected particles is reduced but vacuum fluctuations increase the number of false recordings. When the threshold θ is increased, the number of false recordings decreases, but the number of undetected particles increases. It is intuitively clear that, at some value of the threshold θ, the value must have a minimum (Fig. 1.6.2). Let us find that

$$\frac{dp_{error}}{d\theta} = p(a)W_a(\theta) - p(0)W(\theta) = 0$$

Assuming for simplicity that $p(a) = p(0)$, $a = Const$ we have

$$W_a(\theta) = W_0(\theta),\ W_a(x) = W_0(x-a) \tag{1.6.2}$$

and

$$W_0(\theta) = W_0(\theta - a)$$

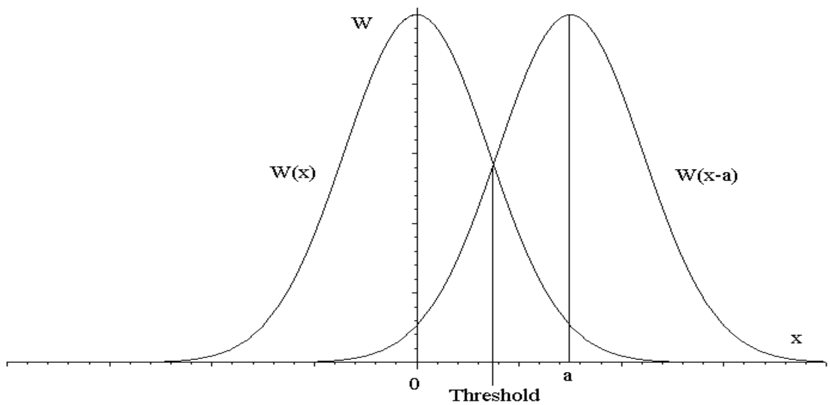

Fig. 1.6.2 *The distribution of the vacuum fluctuations.*

Since $W_0(x)$ is an even function,

$$W_0(\theta) = W_0(a - \theta)$$

hence

$$\theta = \frac{a}{2} = \frac{|\Phi|}{2};\ \ \theta^2 = \frac{1}{4}|\Phi|^2.$$

Consequently, for the optimal quantum detector the threshold energy should be one-fourth of the particle energy. Usually this relation is not hold and inequality is true $\theta^2 << \frac{1}{4} Re^2 \Phi$ or the number of false recording is very high. In compliance with relation (1.6.2) the normalizing condition

$$\int_{-\infty}^{+\infty} W_0(x)dx = 1$$

and by assuming that the flight of the particle or its absence are equiprobable events $p(a) = p(0) = \frac{1}{2}$ expression (1.6.2) can be transformed:

$$p_{error} = \frac{1}{2}\left(\int_{-\infty}^{\frac{a}{2}} W_a(x)dx + \int_{\frac{a}{2}}^{+\infty} W_0(x)dx\right) = \int_{\frac{a}{2}}^{\infty} W_0(x)dx = \frac{1}{2} - \int_0^{\frac{a}{2}} W_0(x)dx$$

After introducing a new variable $y = x/\sigma$, where σ is the r. m. s. of vacuum fluctuations, being normally distributed, we obtain

$$p_{error} = \frac{1}{2} - \int_0^{\frac{a}{2\sigma}} V_0(y)dy,$$

$$V_0(y) = \frac{1}{\sqrt{2\pi}} exp\left[-\frac{y^2}{2}\right].$$

Thence,

$$p_{error} = \frac{1}{2} - \frac{1}{\sqrt{\pi}} \int_0^{\frac{a}{8\sigma}} exp\left(-z^2 dz\right) = \frac{1}{2}\left(1 - erf\sqrt{\frac{a^2}{8\sigma^2}}\right)$$

Then the error of the detectors is small and expressed as a fraction of the form

$$p_{error} = 10^{-P}$$

where $P = 0 \cdot\cdot 6$ for most existing instruments. Denoting $\rho = \dfrac{a^2}{\sigma^2}$ we have the probability of detecting the particle, if it exists, in the form

$$P = -log\frac{1}{2}\left(1 - erf\sqrt{\frac{\rho}{8}}\right) = -log\frac{1}{2}\left(1 - erf\frac{Re\,\Phi}{\sqrt{8\sigma^2}}\right)$$

This is the interpretation of a wave function in unitary quantum theory. The relation $P(\rho)$ does not make an impression until a plot of $P(\rho)$ is seen which is well approximated, in a wide range

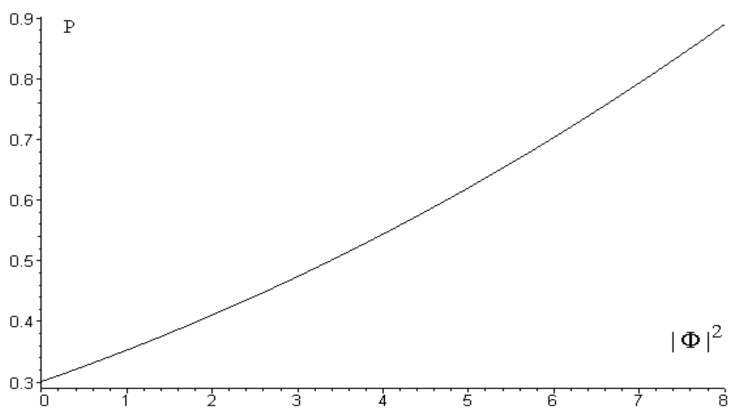

Fig. 1.6.3 *Probability of regular detection of particle as a function of* $|\Phi|^2$

as a straight line (Fig. 1.6.3). In ordinary quantum mechanics it is postulated that $P = \Psi^*\Psi$, but nothing is said about the kind of detectors that are used for the measurement. In unitary quantum mechanics the statistical interpretation is obtained from the mathematical formalism of the theory. The latter includes the consideration of the problem of the statistical interaction between the particle and the detector and the sensitivity of the latter is accounted for.

Since $\rho \approx |\Phi|^2$ and in the ordinary formulation of quantum mechanics $P = \Psi^*\Psi$ then $|\Phi|^2$ and $\Psi^*\Psi$ are seen to coincide with an accuracy of terms of the second order. This correction can be verified experimentally as deflections that appear in the contrast of interference and diffraction pictures should be visible. The position of maxima and minima in such pictures cannot, of course, be affected. The most enterprising experimentalists who want to see the light at the end of the tunnel will hopefully check this.

We can easily paraphrase A. Einstein's words about *"God playing dice"*. Now it is quite evident that God does not play each quantum event creating that or another vacuum fluctuation with only one aim: To force the Geiger counter to detect the particle. It is not so absolutely clear if God can do it at all, because for this He should be able to tug at all the threads all over the Universe and moreover He would need an Ultra-Super-Computer. Apparently God is a perfect mathematician, for He knows Alexander Lyapunov's Central Limit Theorem. That is why He may have decided to make a simple normal distribution of vacuum fluctuations caused by vanishing particles all over the Universe.

Two questions remain, however: Was it God who created that Chaos and how did He manage to do it?

1.7 The Connection of UQT Equations with Telegraph Equations

Just look how magic is the world,

Philosophize, your mind be turned

A. Griboyedov

It is known that the current and tension of alternating electric current in pare lines satisfy the telegraph equation that was definitely derived for the first time by Oliver Heaviside from the Maxwell equation. That equation is a relativistic non-invariant which nevertheless lets us see how it corresponds to Quantum mechanics. The question is that the main relativistic relation between energy, impulse, and mass

$$E^2 = P^2 + m^2 \tag{1.7.1}$$

is still beyond any doubt. In particular, all of the previous paragraphs are based on relativistic invariance. Nevertheless, we shall ask ourselves once again about what will happen with that relation at the exact moment if the wave packet disappears being spread over the space. At that moment the particle will not exist as a local formation. This means that in the local sense there is no mass, local impulse, or energy. The particle in that case, within sufficiently small period of time, is essentially non-existent, for it does not interact with anything. Perhaps this is why the relation (1.7.1) is average and its use at the wavelength level is equal or less than the De Broglie wavelength, which is just illegal. The direct experimental check of that relation at small distances and short intervals is hardly possible today. If the relation (1.7.1) is declined, then it may result in an additional conservation of energy and impulse refusal; but, as we know, according to the Standard Quantum Theory, that relation may be broken within the limits of uncertainty relation. On the other hand, the Lorenz’s transformations have appeared when the transformation properties of Maxwell’s equations were analyzing. However electromagnetic waves derived from solutions of Maxwell’s equations move all in vacuum with the same velocity, i.e. are not subjected to dispersion and do not possess relativistic invariance. Our partial waves, that form a wave packet, is identified with a particle, possess always the linear dispersion. Under such circumstances, it would be quite freely for authors to spread the requirement of relativistic invariance to partial waves. Such requirement has sense in respect only to wave packet's envelope, which appears if we observe a moving wave packet and his disappearance and reappearance. May be the origin of relativistic invariance would be connected in future with the fact that an envelope remains fixed in all inertial reference frames; only the wave's length is changed.

If we resigned the relativistic invariance, then we get quite simple relativistic non-invariant second-order equations for the wave packet in scalar field, viz., the telegraph equation. At first, let us examine the telegraph equation and some of its properties. It looks like the following:

$$\frac{\partial^2 \Upsilon(x,t)}{\partial x^2} = LC\frac{\partial^2 \Upsilon(x,t)}{\partial t^2} + (RC+GL)\frac{\partial \Upsilon(x,t)}{\partial t} + GR\Upsilon(x,t)\,, \qquad (1.7.2)$$

where $\Upsilon(x,t)$ is the current and the tension on line is within x range from some fixed point, and values C, R, L, G - are capacity, active resistance, inductance, and line escaping isolation accordingly.

Let us introduce the next more suitable relations:

$$a_0 = LC\,,\quad 2b_0 = RC+GL\,,\quad c_0 = GR\,.$$

Then the equation will be following:

$$\frac{\partial^2 \Upsilon(x,t)}{\partial x^2} = a_0\frac{\partial^2 \Upsilon(x,t)}{\partial t^2} + 2b_0\frac{\partial \Upsilon(x,t)}{\partial t} + c_0\Upsilon(x,t) \qquad (1.7.3)$$

After transformation

$$exp(b_0 t)Y(x,t) = u(y,z) \quad y = x+ct\,,\quad z = x-ct\,)$$

equation (1.7.3) results in

$$\frac{\partial^2 u}{\partial y \partial z} + \lambda u = 0\,, \qquad (1.7.4)$$

where

$$\lambda = \frac{(RC - GL)^2}{16LC}.$$

That equation belongs to the class of second-order hyperbolic equation. The most important role in its solution plays the Bernhard Riemann function. In the case of equation (1.7.4) this function has the form:

$$R(y,z) = \mathrm{J}_0\left(\sqrt{4\lambda(y-\xi)(z-\eta)}\right),$$

where $J_0(v)$-Bessel function. However, if we introduce function:

$$\Upsilon(x,t) = \exp\left(-\frac{b_0}{a_0}t\right)\Psi(x,t), \tag{1.7.5}$$

then equation (1.7.3) results in the Klein-Gordon equation

$$\frac{\partial^2\Psi(x,t)}{\partial t^2} - V^2\frac{\partial^2\Psi(x,t)}{\partial x^2} - M^2\Psi(x,t) = 0, \tag{1.7.6}$$

where

$$V = \frac{1}{\sqrt{a_0}}, \quad M = \frac{\sqrt{b_0^2 - a_0 c_0}}{a_0}.$$

O. Heaviside obtained the condition under which linear signal propagation would be free from distortion:

$$\frac{G}{C} = \frac{R}{L}$$

If using this relation, equation (1.7.6) may be written as a simple wave equation:

$$\frac{\partial^2 U(x,t)}{\partial t^2} = W^2 \frac{\partial^2 U(x,t)}{\partial x^2},$$

where

$$W = \frac{1}{\sqrt{LC}}.$$

Using the general solution of wave equation and also (1.7.5), we obtain the general solution of telegraph equation [165, 166, 200, 201]:

$$Y(x,t) = \exp\left(-\frac{R}{L}t\right)\left[\varphi(x - Wt) + \phi(x + Wt)\right], \tag{1.7.7}$$

where $\varphi(x - Wt)$ and $\phi(x + Wt)$ are arbitrary functions. Now it is easy to see that solution of (1.7.7) type may be considered as a wave packet running in opposite directions and periodically modulated with an exponential factor on condition that its index is imaginary. As a result, the considered solutions of (1.7.7) type give us an opportunity to look for an analogy between UQT and telegraph equations. As a matter of fact, such an analogy is physically suggested itself as far as in the long Lecher wire in standing-wave mode there exist periodical with the wavelength points. This may be either short-circuited or blocked because of either the current or tension equal to zero (points, where packets vanish). This can be experimentally carried out in a perfect way. Usually such an experiment is a lecture-demonstration for the students of universities.

Expression (1.7.7), in the case of periodical vanishing and appearing of wave packet (UQT new wave function), taking into account mass oscillation, may be rewritten in the form:

$$F(x,t) = \exp\left(i\frac{mv^2}{\hbar}t\right)\left[\varphi(x - vt) + \phi(x + vt)\right], \tag{1.7.8}$$

where packets running in both positive and negative directions $\varphi(x,t)$ and $\phi(x,t)$ are totally arbitrary. For function $F(x,t)$ telegraph equation can be written in the form:

$$\frac{\partial^2}{\partial x^2}F(x,t)-\frac{1}{v^2}\frac{\partial^2}{\partial t^2}F(x,t)+2i\frac{m}{\hbar}\frac{\partial}{\partial t}F(x,t)+\frac{m^2v^2}{\hbar^2}F(x,t)=0 \qquad (1.7.9)$$

Equations resembling (1.7.9) may be obtained from Maxwell equations by making a supposition about imaginary resistance of the conductor and using Oliver Heaviside reasoning while deriving from the telegraph equation (1.7.2). However, the equation (1.7.9) has another solution matching the UQT main idea about the appearing and vanishing packet. That solution [1] has the following form:

$$F(x,t)=\exp\left(\pm i\frac{mv}{\hbar}x\right)\varphi(x\mp vt) \qquad (1.7.10)$$

where we should take the top or bottom sign. Let us write function (1.7.8) or (1.7.10) in the form:

$$F(x,t)=\exp\left(i\frac{mv^2}{\hbar}t\right)\Psi(x,t) \qquad (1.7.11)$$

or

$$F(x,t)=\exp\left(i\frac{mv}{\hbar}x\right)\Psi(x,t) \qquad (1.7.12)$$

By substituting function (1.7.11) into the equation (1.7.9) we get

$$\exp\left(i\frac{mv^2}{\hbar}t\right)\left(v^2\frac{\partial^2}{\partial x^2}\Psi(x,t)-\frac{\partial^2}{\partial t^2}\Psi(x,t)\right)=0$$

Reducing the exponential function we get the wave equation. So in the new quantum equation (1.7.9) O. Heaviside conditions are automatically satisfied (absence of distortion in telegraph equation solution).

Let us insert in our equation (1.7.9) potential U(x) in a general way (here we get an unsolved problem as far as the particle is oscillating and her parameters are changing, but let us temporarily shut our eyes). The velocity of the particle with the energy E in a field with potential $U(x)$ may be written as follows:

$$v = \sqrt{\frac{2\left(E - U(x)\right)}{m}}$$

Substituting it into the equation (1.7.9) and rejecting imaginary terms, we get:

$$\left[-2\hbar^2 E \frac{\partial^2}{\partial x^2} + 2\hbar^2 U(x)\frac{\partial^2}{\partial x^2} + \hbar^2 m \frac{\partial^2}{\partial t^2} - 4mE^2 + 8mEU(x) - 4mU(x)^2\right] F(x,t) = 0 \qquad (1.7.13)$$

Let us divide variables in the equation (1.7.13) in accordance with the standard Fourier technique, assuming that

$$F\left(x,t\right) = \Psi(x)T(t)$$

After a common substitution in (1.7.13) and dividing by the product of sought functions we get:

$$\frac{\hbar^2}{\Psi(x)}\left(U(x) - E\right)\frac{\partial^2 \Psi(x)}{\partial x^2} + \frac{m\hbar^2}{2T(t)}\frac{\partial^2 T(t)}{\partial t^2} - 2mE^2 + 2mU(x)\left(2E - U(x)\right) = 0 \qquad (1.7.14)$$

After coordinate function $\Psi\left(x\right)$ separation and after simple transformations we get the following equation

$$\frac{U(x) - E}{\Psi(x)}\left[2mU(x)\Psi(x) - 2mE\Psi(x) - \hbar^2 \frac{\partial^2 \Psi(x)}{\partial x^2}\right] = 0$$

and we obtain easily the Schroedinger equation:

$$\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\Psi(x)=\big(U(x)-E\big)\Psi(x)$$

Now substitute function (1.7.12) into equation (1.7.9). We obtain

$$\exp\left(i\frac{mv}{\hbar}x\right)\left[-2imv^3\frac{\partial\Psi}{\partial x}-\hbar v^2\frac{\partial^2\Psi}{\partial x^2}+\hbar\frac{\partial^2\Psi}{\partial t^2}-2imv^2\frac{\partial\Psi}{\partial t}\right]=0\,.$$

By rejecting imaginary terms and reducing we get the wave equation and Heaviside conditions for the absence of distortion are again satisfied. It is curious that while rejecting imaginary terms and requiring $v\to c$, equation (1.7.9) is automatically transformed into the Klein-Gordon type equation. All previous reasoning can be easily generalized into a three-dimensional case.

The obtained results are quite amazing. It is well known that nearly any equation of theoretically non-quantum physics can result from Maxwell equations. That is why Ludwig Boltzmann said about Maxwell equations: *"It is God who inscribed these signs, didn't He?"* Modern science has changed not a semi-point in these equations, and now it appears that even non-relativistic quantum mechanics in the form of the Schroedinger equation may also be extracted from the Maxwell equation. The same can be said about the Klein-Gordon relativistic equation.

It is possible to write down (for the invariance-lover) the following two variants of our telegraph equations:

$$\frac{1}{v^2}\frac{\partial^2F(x,t)}{\partial t^2}-\frac{\partial^2F(x,t)}{\partial x^2}+\frac{2imc^2\sqrt{1-\frac{v^2}{c^2}}}{\hbar v}\frac{\partial F(x,t)}{\partial x}+\frac{m^2c^4}{\hbar^2v^2}\left(1-\frac{v^2}{c^2}\right)F(x,t)=0 \qquad (1.7.15)$$

and

$$\frac{1}{v^2}\frac{\partial^2 F(x,t)}{\partial t^2}-\frac{\partial^2 F(x,t)}{\partial x^2}-\frac{2imc^2\sqrt{1-\frac{v^2}{c^2}}}{\hbar v^2}\frac{\partial F(x,t)}{\partial t}-\frac{m^2c^4}{\hbar^2 v^2}\left(1-\frac{v^2}{c^2}\right)F(x,t)=0 \qquad (1.7.16)$$

These two equations are satisfied exactly by relativistic invariant solutions in the form of a standard planar quantum-mechanical wave and also in the form of disappearing and appearing wave-packet, viz.

$$F(x,t)=exp\left(\frac{i}{\hbar}\frac{mc^2t-mvx}{\sqrt{1-\frac{v^2}{c^2}}}\right)$$

$$F(x,t)=exp\left(\frac{i}{\hbar}\frac{mc^2t-mvx}{\sqrt{1-\frac{v^2}{c^2}}}\right)\phi(x-vt)$$

This circumstance is extremely striking, but the analysis of our equations is very complicated and we will leave it aside now.

The next natural step is an attempt to calculate mass spectrum for scalar particles: for the spherically symmetric case, the Schroedinger equation, after angle and radial variables separation, is (Plank constant $\hbar=1)$:

$$\frac{\partial^2 f(r)}{\partial r^2}+\frac{2}{r}\frac{\partial f(r)}{\partial r}+8\pi mf(r)-\frac{L(L+1)}{r^2}f(r)=0\,, \qquad (1.7.17)$$

where L takes the value 0, 1, 2, 3, 4…

The general solution of this equation may be expressed with the help of Bessel functions, which has the following form:

$$f(r) = \frac{C_1 J_{L+\frac{1}{2}}(2\sqrt{2\pi mr})}{\sqrt{r}} + \frac{C_2 Y_{L+\frac{1}{2}}(2\sqrt{2\pi mr})}{\sqrt{r}} \tag{1.7.18}$$

Now we can calculate the particle mass as an integral of packets Module Square over infinite range:

$$m = \frac{4\pi}{c^2} \int_0^\infty \left| f(r) \right|^2 r^2 dr \tag{1.7.19}$$

Note that we have not be able to do it before, as according to the standard quantum theory, the particles are not considered as wave packets.

After substitution of solution (1.7.18) into (1.7.17) we obtain the equation for different masses (mass spectrum) at different values of L. Unfortunately, integral (1.7.19) diverges either at null or infinity, and all masses result as infinite for every value of constants C_1, C_2. The reason of it lies in wrong choice of the class of the decisions of the equation Schrodinger. The causes of divergences that worried quantum physics nearly one century ago remain obscure, including using the approach described in this section.

1.8 Elementary Particle Mass Spectrum Within Unitary Quantum Theory

There at unknown paths,

The tracks of mysterious beasts are…

A. S. Pushkin

The mass spectra problem of elementary particles in a standard quantum theory currently faces a number of insurmountable obstacles, as it is in fact absolutely unclear how to set and solve such a problem from the conceptual viewpoint.

Within the Unitary Quantum Theory (UQT), this problem is simpler to some extent: Elementary particles are viewed as stable wave packets, which while moving, retain their dimensions and shapes, but periodically emerge and disappear at the de Broglie wavelength [162, 164].

As is commonly known in a non-linear media case, the influence of non-linearity and dispersion is destructive. In a general case, the former deform, distort, and smear out over space this localized wave-packet-type formation. Yet for some kinds of wave packets there may be an unstable balance between the contradictory effects of non-linearity and dispersion, which leads to the existence of stable wave packets (particles) and the electric charge quantizing. It is only natural that such a balance is valid only for some specified types of dispersion equations, non-linearity, and wave packets. Further on it will be shown that in our model the number of such packets may be rather numerous, but always limited.

We will show that Eq. (1.7.16) (considered in the case of 3-dimension coordinate space (r,θ,φ)) allows, namely, to determine theoretically the mass spectrum of elementary particles.

Such equation for the function $u=u(r,\theta,\varphi)$ is following:

$$\frac{1}{v^2}\frac{\partial^2 u}{\partial t^2}-\frac{1}{r^2 sin\theta}\left(2r\,sin\theta\frac{\partial u}{\partial r}+r^2 sin\theta\frac{\partial^2 u}{\partial r^2}+cos\theta\frac{\partial u}{\partial \theta}+sin\theta\frac{\partial^2 u}{\partial \theta^2}+\frac{1}{sin\theta}\frac{\partial^2 u}{\partial \phi^2}\right), \quad (1.8.1)$$
$$-\frac{2iMc^2\sqrt{1-\frac{v^2}{c^2}}}{v^2\hbar}\frac{\partial u}{\partial t}-\frac{M^2c^4}{v^2\hbar^2}(1-\frac{v^2}{c^2})u=0$$

(the symbol m is replaced by M).

We will use the natural system of units and put $\hbar = 1, \mathrm{c} = 1$, and will seek the solution of Eq. (1.8.1) inthe following form:

$$u = \frac{f}{r}\exp\left(\frac{iMt}{\sqrt{1-v^2}} - \frac{iMvr}{\sqrt{1-v^2}}\right), \tag{1.8.2}$$

where $f = f(r,\theta,\varphi)$ is some function not depending on t. This function represents as hardened wave packet in coordinate space (r,θ,φ). Substituting (1.8.2) in Eq. (1.8.1), we get

$$\begin{aligned} &2iMvr^2 cos^2\theta\frac{\partial f}{\partial r} - 2iMvr^2\frac{\partial f}{\partial r} + r^2\sqrt{1-v^2}\frac{\partial^2 f}{\partial r^2}sin^2\theta \\ &+\sqrt{1-v^2}\frac{\partial^2 f}{\partial\theta^2}sin^2\theta + \sqrt{1-v^2}\left(\frac{\partial^2 f}{\partial\phi^2} + sin\theta cos\theta\frac{\partial f}{\partial\theta}\right) = 0 \end{aligned}. \tag{1.8.3}$$

We will seek the solution of Eq. (1.8.3) in form:

$$f = R(r)Y_{Lm}(\theta,\varphi), \tag{1.8.4}$$

where

$$\mathrm{Y}_{\mathrm{Lm}}(\theta,\phi) = \frac{\sqrt{(2\mathrm{L}+1)(\mathrm{L}-\mathrm{m})!}}{2\sqrt{\pi(\mathrm{L}+\mathrm{m})!}}\mathrm{P}_{\mathrm{L}}^{\mathrm{m}}(\cos\theta)\exp(\pm\mathrm{i}\mathrm{m}\phi), \tag{1.8.5}$$

$P_L^m(\cos\theta)$ is the Legendre function, $Y_{Lm}(\theta,\varphi)$ is the Spherical Harmonic and L,m are nonnegative integers L=0, 1, 2, 3,…, $\mathrm{m} = 0 \pm 1 \pm 2 \pm 3$.. besides $m \le L$. Substituting (1.8.4) in Eq. (1.8.3), we come to the following equation with respect to the function $R(r)$:

$$\left(\frac{d^2R(r)}{dr^2}r^2\sqrt{1-v^2} - 2i\frac{dR(r)}{dr}Mvr^2\right) - R(r)L^2\sqrt{1-v^2} - R(r)L\sqrt{1-v^2} = 0 \tag{1.8.6}$$

The solution $R(r) = R_L(r)$ of this equation depends on parameter L and we obtain the family of solutions $u_{Lm}(r,\theta,\varphi,t)$ of equation (1.8.3) depending on parameters L,m and describing corresponding partial wave-packets. It is natural to suppose that the modulus of every solution u_{Lm} describes the amplitude of the world unitary potential Φ_{Lm} determined by this equation, and the world potential itself is represented by the quadrate of amplitude modulus, i.e.

$$\Phi_{Lm} = |u_{Lm}|^2 = \left|\frac{R_L(r)}{r} Y_{Lm}(\theta,\varphi)\right|^2. \tag{1.8.7}$$

Further, we consider the gradient of this potential as the tension of corresponding field (it is the custom in electrodynamics) of the partial wave packet and consider the quadrate of the tension as the density W_{Lm} of the energy or of the wave packet's mass distributed continuously in space. So, the mass $M = M_{Lm}$ of our partial wave packet may be determined as the integral of density W_{Lm} over all space (r,θ,φ):

$$M = \int_0^{\infty}\int_0^{\pi}\int_0^{2\pi} W_{Lm}\, r^2 \sin\,\theta dr d\,\theta d\varphi, \tag{1.8.8}$$

where $W_{Lm} = |\text{grad}\,\Phi_{Lm}|^2$. We rewrite the equation (1.8.6) in form:

$$2ivM = \frac{1}{r^2 R'(r)}(R''(r)r^2 - L(L+1)R(r))\sqrt{1-v^2}, \quad (' = \frac{d}{dr}). \tag{1.8.9}$$

We consider the mass of the wave packet as its inner (proper) characteristic not depending on the velocity of its movement. So, we set $v = 0$ and obtain the following differential equation for $R(r)$:

$$R'' - \frac{L(L+1)}{r^2} R = CR', \tag{1.8.10}$$

where C is some constant. This equation possesses the analytical general solution:

$$R(r, C_1, C_2) =$$

$$C_1 \exp(\frac{C}{2} r)\sqrt{r}\, J(L + \frac{1}{2}, \frac{1}{2}\sqrt{-C^2} r) + C_2 \exp(\frac{C}{2} r)\sqrt{r}\, Y(L + \frac{1}{2}, \frac{1}{2}\sqrt{-C^2} r), \tag{1.8.11}$$

where $\tilde{N}_1, C_2$ arbitrary constants and J and Y are the Bessel functions. Since we seek the finite solution $R(r)$ for $r \to 0, r \to \infty$ and tending to zero for, $r \to \infty$ we set $C_2 = 0$ and can set some positive value for C_1 and some negative value for the constant C in Eq. (1.8.11). The calculations show the choice of these constants has influence only on the absolute value of the masses calculated below but the ratios of these masses remain the same. We have chosen the simplest values

$$C_1 = 1, C = -2$$

and have obtained following solution

$$R(r) = \sqrt{r} \exp(-r) \mathrm{J}(L + \frac{1}{2}, ir)), \tag{19.12}$$

where $\mathrm{J}(L + \frac{1}{2}, ir)$ is the Bessel function of 1st type with imaginary argument, or

$$R(r) = i^{L+\frac{1}{2}} \sqrt{r} \exp(-r) \mathrm{I}(L + \frac{1}{2}, r), \tag{1.8.13}$$

where $\mathrm{I}(L+\frac{1}{2},r)$ is the modified Bessel function of 1st type. So, we obtain the following expression for the world unitary potential Φ_{Lm} (taking into consideration (1.8.2; 1.8.4; 1.8.5; 1.8.7) :

$$\Phi_{Lm}=\frac{e^{-2r}}{4\pi r}\left|\frac{(2L+1)(L-m)!\,\mathrm{I}(L+\frac{1}{2},r)^2\,\mathrm{P}_L^m(\cos\theta)^2}{(L+m)!}\right| \tag{1.8.14}$$

Now, we form $\operatorname{grad}\Phi_{Lm}$ considered as the tension of the field and form also the quadrate of its modulus considered as the mass density W_{Lm}. We obtain:

$$W_{Lm}=2e^{-4r}\left(\frac{(L-m)!^2\,\mathrm{I}(L+\frac{1}{2},r)^2\left((L+r+1)\,\mathrm{I}(L+\frac{1}{2},r)-r\,\mathrm{I}(L-\frac{1}{2},r)\right)^2\mathrm{P}_\mathrm{L}^\mathrm{m}(\cos\theta)^4(L+\frac{1}{2})^2}{\pi^2r^4(L+m)!^2}+\right.$$

$$\left.+\frac{(L+\frac{1}{2})^2\,\mathrm{I}(L+\frac{1}{2},r)^4(L-m)!^2\,\mathrm{P}_\mathrm{L}^\mathrm{m}(\cos\theta)^2\left((m-L-1)\,\mathrm{P}_{L+1}^\mathrm{m}(\cos\theta)+(L+1)\cos\theta\,\mathrm{P}_\mathrm{L}^\mathrm{m}(\cos\theta)\right)^2}{\pi^2r^4(L+m)!^2\sin^2\theta}\right) \tag{1.8.15}$$

The integrals of W_{Lm} over all spherical space (r,θ,ϕ) for different $\mathrm{L}=0,1,2,\ldots$ and $\mathrm{m}=0,\pm1,\pm2,\ldots,\mathrm{m}\le\mathrm{L}$ is equal to required different masses M_{Lm} of elementary particles, i.e.

$$M_{Lm}=\int_0^\infty\int_0^\pi\int_0^{2\pi}W_{Lm}r^2\sin(\theta)\,dr\,d\theta\,d\varphi \tag{1.8.16}$$

Since W_{Lm} does not depend on φ and the Legendre functions in expressions of W_{Lm} may be integrated analytically, we calculated, at first, analytically (with help of Mathematics-9) the integrals

$$\mathrm{U}_{\mathrm{Lm}} = \int_0^{\pi} \mathrm{W} r^2 \sin(\theta) d\theta \int_0^{2\pi} d\phi = 2\pi \int_0^{\pi} \mathrm{W} r^2 \sin(\theta) d\theta \tag{1.8.17}$$

and then calculated numerically (with the help of Mathematics-9) the integrals

$$M_{Lm} = \int_0^{\infty} U_{Lm} dr \tag{1.8.18}$$

For example, we have obtained for L=0 и m=0:

$$U_{00} = \frac{8e^{-4r}\sinh(r)^2}{\pi^3 r^4}\left\{(r^2 + \frac{1}{2} + r)\cosh(r)^2 - r(1+r)\sinh(r)\cosh(r) - \frac{(1+r)^2}{2}\right\}$$

and

$$\mathrm{M}_{00} = \int_0^{\infty} \mathrm{U}_{00} dr = 0.003944364169$$

For $\mathrm{L} = 1, \mathrm{m} = 1$

$$U_{11} = \frac{8e^{-4r}}{\pi^3 r^8}\left[\left(r^6 + 5r^5 + \frac{93}{8}r^4 + 13r^3 + \frac{61}{4}r^2 + 2r + \frac{17}{8}\right)cosh^4 r -\right.$$

$$-r\,sinh\,r\,cosh^3\left(r^5 + 5r^4 + 11r^3 + \frac{33}{2}r^2 + 8r + \frac{17}{2}\right) -$$

$$-cosh^2 r\left(\frac{1}{2}r^6 + 3r^5 + 10r^4 + 14r^3 + \frac{71}{4}r^2 + 4r + \frac{17}{4}\right) +$$

$$\left. + r\,sinh\,r\,cosh\,r\left(r^4 + 3r^3 + 8r^2 + 8r + \frac{17}{2}\right) + \frac{1}{2}r^4 + r^3 + \frac{5}{2}r^2 + 2r + \frac{17}{8}\right]$$

and

$$M_{11} = 0.00006798678730.$$

The calculations for small values of L are sufficiently simple. But for large L, the quantities U_{Lm} are represented by long polynomials in r and $\cosh(r), \sinh(r)$ with enormous numerical coefficients and the integration of these polynomials meets serious technical difficulties.

We consider the ensemble $\mathrm{L}+1$ particles (masses) with given L and $m=0...\pm L$ to be one family and we will use the notations $\mathrm{M}_{\mathrm{L},0}$, $\mathrm{M}_{\mathrm{L},1}$, ..., $\mathrm{M}_{\mathrm{L},\mathrm{L}}$ for particles (masses) of the family with given L. We have calculated and analyzed in full the masses of 49 families *(L=0,1...48)*,, i.e. of 1225 particles. Our PC with $3\mathrm{GHz}, \mathrm{RAM}=4\mathrm{GB}$ has required for these calculations nearly 3 weeks of computing time. All calculations were checked by Maple-18.

We have compared our theoretical spectrum for 1225 masses with known experimental spectrum for elementary particles measured in MeV. The zero-point for the matching of both spectra was required. We have taken for such matching the quotient of the muon mass to the electron mass. As we know, this quotient for observed muons and electrons is measured experimentally [15] with the most precision and is equal 206.76884(10). Each our calculated mass was divided consecutively by all other 1224 masses and the resulting quotients were compared with the mentioned number. It turned out that the quotient of our masses $\mathrm{M}_{16,10}/\mathrm{M}_{48,45}$ is equal to 206.7607796 (with relative divergence 0.0039%) and we have taken our mass $\mathrm{M}_{48,45}$ equal to 0.2894982442536304 $\cdot 10^{-10}$ for zero-point, i.e. for our electron mass. After, there were divided all other 1224 masses $M_{L,m}$ by $M_{48,45}$ and we have obtained our theoretical spectrum in electron masses which may be compared (after expressing in MeV)

with known experimental masses. Here is the table with our masses M_{Lm} for 34 cases of the well coincidence with well known experimental values (relative errors are less than 1% in 30 cases and between 1.3% and 1.8% in three cases).

Table 1.8.1 *Table of some well known experimental masses of elementary particles.*

$\mathbf{M}_{\mathbf{L,m}}$	**Theory**	**Experiment**	**Notation**	***Error %***
$M_{48,45}$	0.51099906	0.51099906	e	--
$M_{16,10}$	105.6545640	105.658387	μ	*0.0036*
$M_{18,4}$	135.8958708	134.9739	π^0	*0.683*
$M_{23,0}$	137.2902541	139.5675	π^+, π^-	*1.62*
$M_{14,1}$	541.7587460	548.86	η	*1.29*
$M_{7,7}$	894.0806293	891.8	K^{*+}, K^{*0}	*0.25*
$M_{12,1}$	936.3325942	938.2723	p	*0.206*
$M_{10,4}$	957.1290490	957.2	ω	*0.0083*
$M_{9,5}$	1110.473414	1115.63	Λ	*0.462*
$M_{8,6}$	1224.151552	1233	b_1^0	*0.71*
$M_{11,1}$	1271.916682	1270	K^*	*0.14*
$M_{9,4}$	1331.705434	1321.32	Ξ^-	*0.78*
$M_{10,2}$	1378,127355	1382.8	Σ^0	*0.33*
$M_{12,0}$	1524.617683	1520.1	Λ_2	*0.29*
$M_{8,5}$	1549.444919	1540 ± 5	F_1	*0.28*
$M_{7,6}$	1595.510637	1594	ω_1	*0.094*
$M_{9,3}$	1601.282953	1600	ρ'	*0.08*
$M_{6,6}$	1718.917400	1720	N_0^3	*0.06*
$M_{10,1}$	1774.917815	1774	K_3^{*+}	*0.051*

$M_{L,m}$	Theory	Experiment	Notation	*Error %*
$M_{8,4}$	1906.842877	1905	Δ^{+}_{5}	*0.096*
$M_{9,2}$	1965.115639	1950	Δ_4	*0.77*
$M_{11,0}$	2092.497779	2100	Λ_4	*0.35*
$M_{7.5}$	2195.695293	2190	N(2190)	*0.25*
$M_{7,4}$	2818.645188	2820	η_c	*0.048*
$M_{10,0}$	2954.549810	2980	η	*0.85*
$M_{6,5}$	3082.979571	3096	J/ψ	*0.42*
$M_{7,3}$	3543.664516	3556.3	χ	*0.35*
$M_{5,5}$	3687.679612	3686.0	ψ'	*0.04*
$M_{7,2}$	4496.650298	4415	ψ'''	*1.84*
$M_{6,4}$	5642.230394	5629.6	Ξ_b	*0.8*
$M_{5,3}$	9499.927309	9460.32	$\Re$`	*0.41*
$M_{6,1}$	10075.78271	10023.3	$\Re$``	*0.523*
$M_{7,0}$	10533.15222	10580	$\Re$```	*0.442*
$M_{2,2}$	131517	125000-140000	Higgs	
$M_{0,0}$	*6962274*	*?*	*Dzhan*	*?*

(e – electron, μ - muon, π^0 - π -meson, p – proton etc.)

Note, the ratio of our proton mass $M_{12,1}$ and our electron mass $M_{48,45}$ is equal 1832.355 with relative error 0.207% in comparison with well known experimental ratio 1836.152167. Our calculated spectrum containing 169 masses from muon to the heaviest mass approximates also others well known particles and, although the coincidences with experimental data are worse but quite acceptable (with relative divergences not more than several per cent). The mass values for negative m coincides with the mass valued for positive m (antiparticles?).

On the whole, this table shows the striking coincidence of our theoretical values with essential quantity of the known experimental masses and, by no means, such coincidence may be called occasional. The probability of such occasional coincidence is less 10^{-60}. Note, the choice of the nominee for the electron's mass is not unique and may be further calculations of families with $L = 60....100$ would allow obtaining the better result. Our calculated theoretical spectrum contains also the values near to the masses of quarks. The experimental data for quarks are not so precise and are determined in an indirect way. We give the separate table with the calculated and experimental quark masses:

Table 1.8.2 *Table calculated and experimental masses of quarks.*

$M_{L,m}$	**THEORY**	***Experiment***
$M_{38,16}$	5.003455873	*3-7*
$M_{30,25}$	2.75072130	*1.5-3.0*
$M_{20,4}$	94.4251568	95 ± 25
$M_{11,1}$	1271.9166	1250 ± 90
$M_{6,4}$	4300.86662	4200 ± 70
$M_{3,0}$	*179100*	178000 ± 4300

We have carried out also the series of calculations M_{Lm} for L exceeding 48 including L=60. The ratio of maximal $M_{00} = 0.0039443641689$ to minimal $M_{60,60} = 0.3909395521 \cdot 10^{-11}$ is of order 10^9. The ratio of maximal M_{00} to the mass $M_{12,1} = 0.5304640719 \cdot 10^{-7}$ of proton is equal 74400. This number does not contradict the known the experimental data.

Note, the radial function $U_{Lm}(r)$ being the density mass as function of r, is equal zero always for r=0 and for all L, m, and, at first, increases very swiftly on the right from for r=0 and then very swiftly decreases. The plot of $U_{Lm}(r)$ reminds for large L quasi delta-function approaching to coordinates origin as L increases (very simplified analogy is shown on Fig. 1.9.1).

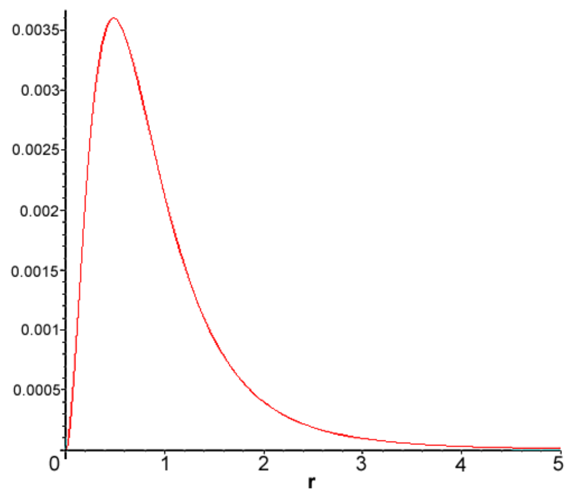

Fig. 1.8.1 *The plot for* $U_{00}(r)$.

Such theoretical model describes a particle as very small bubble in space-time continuum cut by spherical harmonics. Curious, such model, namely, was considered by A. Poincare [161].

Certainly, we do not intend to assert that our results are adequate in full to the known experimental mass spectrum of elementary particles. The divergences are present. Our theoretical spectrum contains the large quantity (1053) of masses between electron mass and muon mass (dark matter?) but such real particles are not observed till now. Our spectrum contains many light particles $M_{L,m}(L > 48)$ with masses differing extremely little one from another. It may be supposed there is exists quasi-continuous distribution of lightest particles not affirmed till now by experiments. We suppose that this region of our calculated spectrum contains also the values corresponding to masses of all 6 neutrinos, and it will be possible to discover their theoretical masses after sufficiently precise experimental determination of their masses.

Our spectrum contains of 169 particles from the muon to the heaviest particle $M_{0,0}$ but there are a large quantity of particles in this interval with short "life-time" (so called "resonances") of order $10^{-22}\,sec$. These divergences require the further researches. With respect to light particles, it may be supposed the existence of some selection principles (not discovered till now theoretically) for such particles and these principles lead to essential decreasing of particles quantity between muons and electrons. We suppose that such principles arise theoretically from some relations between the tensors of different valences (ranks) and spherical functions for different L, m and leave this complicate problem for future researches. May be these light particles constitute the dark matter.

There is a question arose with respect to the particles with short "life-time": may we take all these particles for elementary? Our Unitary Quantum Theory allows formulating the following criterion. *If the way which the particle (which we identify with appearing and disappearing wave packet) passes from the moment of its appearing to the moment of its destruction is much longer than de Broglie wave, then such particle may be called elementary.* Have we reason to call "elementary" the particle with life-time of order $10^{-22}\,sec$?

Let us point to following essential circumstance. Viz., if we use the Schrödinger equation in spherical coordinates (relativistic-noninvariant) or Klein—Gordon equation (relativistic-invariant) instead of our initial equation (5), then we will come to the same theoretical mass spectrum. Really, the mention Schrödinger equation is following:

$$\frac{\hbar^2}{2}\frac{\left(2r\sin\theta\frac{\partial u}{\partial r}+r^2\sin\theta\frac{\partial^2 u}{\partial r^2}+\cos\theta\frac{\partial u}{\partial\theta}+\sin\theta\frac{\partial^2 u}{\partial\theta^2}+\frac{1}{\sin\theta}\frac{\partial^2 u}{\partial\phi^2}\right)}{Mr^2\sin\theta}+i\hbar\frac{\partial u}{\partial t}=0\,, \qquad (1.8.19)$$

where M is the particle's mass. We will seek the solution of this equation in form of unitary wave packet f :

$$u=\frac{f}{r}exp(-i\frac{Mv^2}{2\hbar}t+i\frac{Mv}{\hbar}r) \qquad (1.8.20)$$

where $f = f(r,\theta,\varphi)$ is the function of coordinates and does not depend on the time. The function u is considered as the amplitude of the world unitary potential Φ. Substituting (1.8.20) in (1.8.19), we obtain (after simplification) the following equation

$$\hbar r^2\sin^2\theta\frac{\partial^2 f}{\partial r^2}-2iMvr^2\sin^2\theta\frac{\partial f}{\partial r}+\frac{\hbar}{2}\sin 2\theta\frac{\partial f}{\partial\theta}+\hbar\sin^2\theta\frac{\partial^2 f}{\partial\theta^2}+\hbar\frac{\partial^2 f}{\partial\phi^2}=0\,. \qquad (1.8.21)$$

This equation coincides with our equation (1.8.3) if we put $\sqrt{1-\mathrm{v}^2}$ instead $\hbar$. The further study described above remains without changes.

Let us consider Klein—Gordon equation in spherical coordinates and in natural units system ($c=1,\hbar=1$):

$$\frac{\left(2r\sin\theta\frac{\partial u}{\partial r}+r^2\sin\theta\frac{\partial^2 u}{\partial r^2}+\cos\theta\frac{\partial u}{\partial\theta}+\sin\theta\frac{\partial^2 u}{\partial\theta^2}+\frac{1}{\sin\theta}\frac{\partial^2 u}{\partial\phi^2}\right)}{r^2\sin\theta}-\frac{\partial^2 u}{\partial t^2}-M^2u=0\,, \qquad (1.8.22)$$

where M is the particle's mass. We will seek the solution

$$u = \frac{f}{r} exp\left(\frac{iMt}{\sqrt{1-v^2}} - \frac{iMvr}{\sqrt{1-v^2}} \right), \qquad (1.8.23)$$

where $f = f(r, \theta, \varphi)$ is the function of coordinates not depending explicitly on t. Substituting (1.8.23) in (1.8.22), we obtain the following equation after simplification

$$r^2 \sin^2\theta\sqrt{1-v^2}\frac{\partial^2 f}{\partial r^2} - 2ivr^2 M \sin^2\theta\frac{\partial f}{\partial r} + \sin^2\theta\sqrt{1-v^2}\frac{\partial^2 f}{\partial\theta^2} + \sqrt{1-v^2}\frac{\partial^2 f}{\partial\phi^2} + \frac{\sqrt{1-v^2}}{2}\sin 2\theta\frac{\partial f}{\partial\theta} = 0 . \qquad (1.8.24)$$

This equation coincides in full with our equation (1.8.3) and we will come to the same results.

Here is the table with all our theoretical masses from the muon to the heaviest $M_{0,0}$ (MeV).

Table 1.8.3 *Table all theoretical masses from muon to the heaviest particle with name Dzhan.*

105.655, 105.94, 106.241, 108.291, 108.997, 109.597, 110.133, 112.784, 117.054, 118.136, 120.31, 121.826, 122.664, 125.522, 125.71, 127.187, 127.237, 127.306, 131.445, 133.013, 135.896, 137.29, 142.287, 144.326, 145.96, 147.309, 147.698, 149.62, 149.905, 153.765, 153.827, 159.796, 162.135, 162.192, 165.33, 172.249, 177.091, 178.559, 178.758, 180.585, 180.895, 187.69, 192.661, 192.917, 195.832, 199.852, 203.297, 205.588, 209.097, 218.681, 219.639, 221.135, 224.061, 225.089, 231.432, 231.656, 241.805, 249.092, 252.972, 253.184, 269.993, 270.91, 276.443, 280.151, 281.016, 289.488, 300.299, 301.848, 304.024, 314.364, 318.997, 335.848, 339.955, 341.136, 342.52, 349.235, 357.381, 366.838, 373.402, 402.126, 408.316, 423.36, 423.429, 432.83, 445.413, 459.388, 461.593, 472.253, 504.945, 521.772, 529.951, 531.566, 539.326, 541.759, 560.236, 571.51, 606.559, 619.012, 672.537, 686.757, 705.247, 705.477, 730.141, 738.98, 812.354, 828.374, 866.997, 894.081, 897.982, 915.038, 936.333, 957.129, 996.316, 1110.47, 1135.57, 1137.9, 1224.15, 1271.92, 1331.71, 1378.13, 1524.62, 1549.43, 1595.51, 1601.28, 1718.92, 1774.92, 1906.84, 1965.1, 2092.5, 2195.7, 2334.9, 2557.69, 2818.65, 2906.6, 2954.55, 3082.98, 3543.66, 3687.68, 3832.21, 4300.87, 4315.87, 4496.65, 5642.23, 6026.01, 6570.85, 6666.64, 7358.75, 9219.36, 9499.93, 10075.8, 10533.2, 12941.1, 16897., 18035.6, 18261.3, 25000.7, 28935.4, 33698.9, 36955.4, 54518.8, 71060.4, 87704.5, 131517., 179100., 266419., 601983., 1.20005e6 3.4545e6, 6.96227e7.

So, different initial equations (1.8.1), (1.8.19), (1.8.21) (the last is relativistic invariant and the other two are relativistic non-invariant) lead to the same theoretical mass spectrum. Note the following remarkable fact: the standard theory allows detecting spectra by using always the quantum equations with outer potential and as corollaries to geometric relations between de Broglie waves' length and characteristic dimension of potential function. The quantum equation of our theory does not contain the outer potential and describe a particle in empty free space; the mass quantization arises owing to the delicate balance of dispersion and non-linearity which provides the stability of some wave packets number. It is the first case when spectra are detected by using the quantum equations without outer potential.

In view of all said above, we are bold, nevertheless, to say that our results represent the substantial advancement on the way of solution for the extremely complicated theoretical problem of the mass spectrum for elementary particles and to underline that this advancement is owing to our Unitary Quantum Theory. We hope that further analysis with the help of exact equation (1) of our theory will allow to obtain more precise results.

We would like to propose the name "Dzhan-particle" for our heaviest particle $M_{0,0}$ in honor of the general Air Force RF cosmonaut V. A. Dzhanibekov. As we know, particles with mass of such order are observed in cosmic rays.

Nowadays to confirm SM (Standard Model) one should find a Higgs boson and for this purpose the governments of some countries assigned essential sums for the construction of Large Hadron Collider (LHC). For entire SM the interaction with Higgs field is extremely important, as soon without such a field other particles just will not have mass at all, and that till lead into the theory destruction.

To start with we should note that the Higgs field is material and can be identified with media (aether) as it was in former centuries. But SM authors as well as modern physics have carefully forgotten about it. We would not like to raise here once again the old discussion about it. It's a quite complicated problem and let us leaves it to the next generation.

But another problem of SM has never mentioned before: in the interaction with Higgs field any particle obtains mass. As for Higgs boson itself, it is totally falling out of this universal for every particle mechanism of mass generation! And that is not a mere trifle, such "mismatching" being fundamental fraught with certain consequences for SM.

After Higgs boson discovery nothing valuable for the world will happen except an immense banquet. Of course boson will justify the waste of tens billions of Euros… But even now some opinions in CERN are expressed that probably boson non-disclosure will reveal a series of new breath-taking prospects… and where were these voices before construction, we wonder? But that's not the point! If this elusive particle were the only weakness of SM! To our regret today this theory cannot compute correctly the masses of elementary particles including the mass of Higgs boson. More worse, that SM contains from 20 to 60 adjusting – arbitrary! - parameters (there are different versions of SM). SM does not have theoretically proved algorithm for spectrum mass computation – and no ideas how to do it!

All these bear strong resemblance to the situation with Ptolemaic model of Solar system before appearance of Kepler's laws and Newton's mechanics. This earth-centered model of the planets movement in Solar system at the moment of appearance had required introduction of 40 epicycles, specially selected for the coordination of theoretical forecasts and observations. Its description of planets positions was quite good; but later to increase the forecasts accuracy it had required another 40 additional epicycles…

Good mathematicians know that epicycles are in fact analogues of Fourier coefficients in moment decomposition in accordance with Kepler's laws; so by adding epicycles the accuracy of the Ptolemaic model can be increased too. However that does not mean that the Ptolemaic model is adequately describing the reality. Quite the contrary…

The Unitary Quantum Theory allows computing the mass spectrum of elementary particles without any adjusting parameters. By the way computed spectrum has particle with mass 131.51711 GeV (L=2, m=2). Once desired it can be called Higgs boson, it lies within declared by the CERN+Tevatron mass interval 125-140 GeV expected to contain Higgs boson. CERN promises to obtain more precise mass value by December 2014.

When editing of the book was closed find 3 pentaquarks. The significance of each of these masses is more than 9 standard deviations. One has a mass of 4380±8±29 MeV and a width of 205±18±86 MeV (our theory $M_{9,0}$=4315,87 *MeV* while the second is narrower, with a mass of 4449.8±1.7±2.5 MeV and a width of 39±5±19 MeV (our theory $M_{7,2}$=4496,65), third Θ^+ barion has mass 1522±3 *MeV* (our theory $M_{12,0}$=1524.62 *MeV*). It masses were calculated in 2008! [162,164].

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